# Project Selection: Commitment and Competition

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**Abstract**

We examine project selection decisions of firms constrained in the number of projects they can handle at once. A new project opportunity arises every period. Taking on a project requires a commitment of uncertain duration, preventing the firm from selecting another project in subsequent periods until the commitment ends. In our dynamic game, when two firms are free of commitment, they move sequentially in random order. Symmetric pure strategy Markov perfect equilibria always exist. In equilibrium, the first mover strategically rejects some projects that are then selected by the second mover, even when the value of the project is the same for both firms. A monopolist rejects more projects, and adopts ones of higher average quality compared to the duopolist. Duopolists select too few projects compared to their jointly optimal behavior. We extend the model to allow for externalities, asymmetry, and $n > 2$ firms.

**Keywords:** project selection, search, commitment, Markov perfect equilibrium

**JEL Classification Codes:** L10, L13, D21.

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# 1 Introduction

## 1.1 Motivation

Firms, researchers and government agencies repeatedly select projects (e.g., research and development projects, clients to serve, acquisition of start-ups, and new products). In many situations, firms are constrained in the number of projects they can work on at once, and projects can last for a number of periods. Committing limited resources (e.g., researchers' and other skilled employees' time, physical space, lab equipment) to one project may result in the need to forgo future, perhaps more profitable opportunities. In a strategic environment, firms compete for projects; one firm's decision to select a project can affect its rival's opportunities at present because the selected project is no longer available, and in the future, because the firm commits its resources. In this paper, we study project selection of strategic firms that face uncertainty about the duration of the project to which they commit and about the value of future projects.

We analyze a dynamic, discrete time, infinite horizon game. Every period, a new project opportunity arises and perishes if it was not selected. Firms learn the expected return of the current project, and know the distribution from which future project returns are drawn. Random project duration is captured by a fixed probability that commitment will end and free the firm to select a new project in the following period. When two competing firms are free of commitment, we assume they consider projects sequentially (and the decision not to select a project is irreversible). A period's leader—the first firm to consider the project—is chosen at random. The period's follower can select the project only if the leader did not.

Sequential project selection decisions occur in some markets. For example, pharmaceutical companies repeatedly face project opportunities presented by small biotech firms who negotiate with the pharmaceutical companies sequentially;[1] Editors of competing journals choosing which submissions to accept for publication are approached by authors sequentially; Service providers (e.g., consulting firms, contingent fee lawyers, contractors) are sequentially

[1] See, for example, Kolchinsky (2004) page 56 on negotiation agreements, and http://www.secinfo.com/d12MGs.1n4.htm for a news release describing Micrologixs Biotech Inc. entering an exclusive negotiation period with a pharmaceutical company.

approached by clients, and need to decide whether to take the client, knowing that accepting to serve him requires commitment and that rejecting him may push him to a rival firm. A November 2000 headline in the Chicago Sun-Times read, "Coke backs off bid to buy Quaker Oats: Company's decision could open door for rival PepsiCo." Pepsi later acquired Quaker Oats for its Gatorade sports drink.[2]

In our game, a pure strategy symmetric Markov perfect equilibrium always exists. If the commitment to a project only lasted one period, in a symmetric game, a leader would take on any project with a positive return and a follower would never take on any project. But, when projects can last longer than one period and require commitment of resources, positive return projects may be rejected by both firms. The leader in a period would select any project that has a high enough return. Interestingly, we show that there are projects with intermediate levels of return that are rejected by the period's leader, yet selected by the follower. This is true even though both firms have the same value from the current project, and the leadership position is only guaranteed for the current period. To understand the intuition for this result, consider the lowest value project that the follower would select – the follower is indifferent between selecting and rejecting this marginal project. The leader has the same benefit as the follower from selecting this project, but a higher benefit from rejecting it because if the leader rejects, the follower would select and (unless the project ends) the follower will then be busy in the next period guaranteeing the leader the opportunity to select next period. Thus, a benefit from having a busy rival can arise because firms compete over the same project opportunities.

To examine the effects of competition on project selection behavior in the environment we describe, we compare the outcome of the game with two benchmarks: a decision maker with a capacity of one project, this analysis is similar to earlier work and is included for easy comparison;[3] and a decision maker with a two-project capacity. We find that project

[2] Lazare, L. Chicago Sun-Times, November 22, 2000. http://www.highbeam.com/doc/1P2-4568251.html. Informal conversation with a former Coca Cola employee suggested that introducing a new drink requires significant commitment of resources on the part of the company (e.g., market research, bottle design and marketing efforts).

[3] The analysis of a decision maker is similar to labor search models (e.g., McCall, 1970 and Mortensen, 1970), and option value models of investment (e.g, Dixit and Pindyck, 1994).

selection thresholds are lower under competition. That is, a duopolist selects projects that it would have rejected absent a competitor in the market. One explanation for this is that the duopolist expects an inferior selection of projects compared with the monopolist (because his rival will select some of the high value projects). This reduces the value of waiting and makes the duopolist more likely to select the current project. We also compare equilibrium selection with the optimal choice of a joint venture (maximizing sum of profits with a two-project capacity). We find that in duopoly competition, firms reject too many projects compared with the jointly optimal behavior. Intuitively, the joint venture is more flexible in using the capacity than the duopoly and the benefit from a busy rival, which is important in the duopoly situation, is not a consideration for the joint venture.

Our main model assumes that the leader in each period is chosen at random. This assumption is natural when firms are ex-ante symmetric. It illustrates the benefit from having a busy rival, in a setting where it seems most surprising that one firm would accept a project rejected by the other. However, in some cases, one of the firms may have an advantage (perhaps it is better known, or more easily accessible). We study a version of the game in which one firm is the leader in every period. In this asymmetric game, the follower has a lower threshold for accepting projects than the leader for an additional reason: project opportunities for the follower arise from an inferior (right censored) distribution of project returns, because when free, the leader selects the most promising projects.

Our results are robust to the inclusion of payoff externalities. Considering the effect of externalities on project selection thresholds (in the case of long duration projects), we show that firms have lower thresholds (select more projects) under positive payoff externalities than without externalities, and higher thresholds with negative externalities. One might expect positive externalities to create an incentive to reject more projects when the other firm is free. However, the benefit from having a busy rival is reduced with positive externalities, and the concern about committing resources is lower, so that the firm would have less reason to reject projects. We also extend the duopoly game to an oligopoly.

## 1.2 Background Literature

Our paper contributes to the literature on investment in strategic environments. In a deterministic setting, Fudenberg and Tirole (1985) show that strategic firms adopt a new technology with decreasing costs sooner than they would in a monopoly market structure. In their model, firms adopt preemptively to prevent or delay adoption by their opponents, while in our model, a firm could benefit when its opponent selects a project. Weeds (2002), Grenadier (2002), Pawlina and Kort (2006), Mason and Weeds (2010) offer real options models of strategic investment under uncertainty. The first two papers have symmetric models. Pawlina and Kort (2006) include asymmetry in investment costs, and in Mason and Weeds (2010) the leader and the follower have different flow profits. In our model, the firms are identical, but sequential moves effectively add some asymmetry.

Weeds' (2002) duopoly patent race has two sources of uncertainty: first, the value of the patent evolves stochastically; second, success in research has a Poisson arrival. Irreversible investment and uncertainty create an option value for delay in investment, but since only the first to succeed wins the prize, preemption creates an incentive to invest early. In her paper, the position of a first mover is endogenously determined. In our model, a temporary leader position arises exogenously, the advantage to moving first results from the ability to select high return projects and to commit to not pursue lower return ones. In Weeds' (2002) model, only asymmetric equilibria exist for some parameter values, while in our model a symmetric equilibrium always exists, but asymmetric equilibria do not necessarily exist.

Grenadier (2002) introduces a model in which each firm has an option to invest in expanding capacity at any point in time. Market price depends on a stochastic demand shock and on the total quantity firms produce. Grenadier (2002) shows that competition leads firms to invest sooner as the fear of preemption erodes the option value of investment. What drives this is that a firm's profits decline with the quantity produced by its competitors, creating an incentive to invest before others. In equilibrium, all firms invest at the same time and do so sooner than a monopolist would. In our model too, the thresholds for project selection are lower in the duopoly game than with a monopoly. This is because firms compete for the same projects. In Grenadier's model, a firm becomes worse off when its opponent invests, while in our model a firm benefits when its opponent has invested and his resources are committed. Also, in Grenadier's model, investing in the current period does not preclude a firm from investing again in the future. While in our model, when a firm invests, it is committed and will be unable to select future projects until the commitment to the current project ends.

Our model also relates to a surprisingly small number of strategic search models. Reinganum (1982, 1983a) considers a game where sequential search for a technology is undertaken simultaneously. Once all firms have completed their search, they compete in the goods market. The dates at which the firms stop searching play no role. Some authors offered variations and extensions to Reinganum's strategic search models (e.g., Lippman and Mamer, 1993; Taylor, 1995; Daughety and Reinganum, 2000; Hoppe, 2000). In these models, the agents search in different pools. The interaction comes from the effect that the other agent's search outcome has in a subsequent stage. Our model differs from this in that a firm's project selection strategy affects the distribution of projects faced by the other firm. Additionally, our firms alternate between periods of search and periods of development. This is a common setting in job search models, but, to the best of our knowledge, it has not yet been analyzed in a strategic search game.[4]

In the management literature, Cassiman and Ueda (2006) study the decision of an established firm that generates innovations to commercialize either internally, or in an external venture. We compare a version of our model with their paper; in that version, there is a dominant firm (that always has priority in selection). One of their findings is that the innovations commercialized by the established firm have "lower profitability than innovations commercialized through external ventures." In contrast, our dominant firm selects higher value projects. The difference in results are driven by many differences in our models, for example, in Cassiman and Ueda the established firm has more resources and different commercialization capacity while in our model a project generates the same value whether it is

[4]Other models of investment under uncertainty include patent races (e.g., Loury, 1979; Lee and Wilde, 1980; Reinganum, 1983b), search intensity is the main strategic variable in these models. Thomas (2013) analyzes a model of strategic experimentation with a negative externality. Our results are not directly comparable as the models differ significantly.

selected by the dominant firm or the other firm. Another important difference is that they consider Nash bargaining, which gives the established firm an incentive to raise total surplus, while we take a non-cooperative game approach. As Cassiman and Ueda note, related empirical evidence is limited. Gompers and Lerner (2000) compare corporate venture capital investments (which one might interpret as investment by dominant firms) to investments by traditional private venture capital. Examining empirical evidence, they find that "far from being failures, corporate venture investments in entrepreneurial firms appear to be at least as successful [...] as those backed by independent venture organizations." This seems to be more consistent with our prediction, although additional or alternative explanations may be responsible for the observations in their empirical study.[5]

The rest of this paper proceeds as follows. In Section 2, we describe the basic model with a single decision maker. Section 3 develops the game and analyzes the strategic interaction between two firms. In Section 4, we discuss some extensions, including a dominant firm, an oligopoly with $n$ firms, and externalities. Section 5 concludes. Proofs of all the propositions are provided in the appendix.

# 2 Basic Model

We begin by describing the model in the context of a decision maker – a single firm that repeatedly decides whether to select projects that arise sequentially. This case serves as a benchmark for comparison with the game in Section 3. Consider a discrete time infinite horizon model. The firm maximizes the discounted sum of expected payoffs with a discount factor $0 < \delta < 1$. A new project opportunity arises every period. If the firm is not currently committed, it decides whether to select the current project. A project requires a commitment

[5] Our work is also related to literature on queuing. Lippman and Sheldon (1971) characterized the optimal strategy of a single server who faces clients with different rewards and expected service times. Kalai, Kamien and Rubinovitch (1992) examined strategic interactions between servers. Our problem has also some similarity to the problem of dynamic assignment of a single durable object to successive agents, considered by Bloch and Houy (2012).

of resources, which prevents the firm from working on more than one project at a time.[6] Binding commitments to a project can arise due to agreements with clients, employees, or suppliers, or because the firm cannot search for new opportunities while working on the current project. It is also possible that projects require a sunk cost that makes abandoning a project, even for a better one, not worthwhile.

The duration of projects is uncertain. In each period, the commitment to a project will end by the next period with a probability $p \in [0, 1]$. For example, for a service provider, project duration can be the time it takes to complete the service; for a firm engaged in acquisitions, the time it takes to transfer knowledge from the innovator to the firm, to develop the product, and to come up with a marketing strategy.

Each project has a randomly drawn return $v$, which is the expected discounted present value of net benefits from the project at the time it is selected. Project returns are identically and independently drawn from a known distribution with a cumulative distribution function $F(v)$ on a finite support $[\underline{v}, \overline{v}]$, such that $\underline{v} < 0$, and $\overline{v} > 0$. For all $v$, $F$ is differentiable with a finite density $f(v) > 0$. The expected value of returns is positive, $\int_{\underline{v}}^{\overline{v}} v f(v) dv > 0$. The payoff in a period in which no project is selected is zero.

We assumed the payoff $v$ is obtained immediately when the project is selected. With our assumption of a binding commitment, this value could stand for an expected value of receiving the prize at the end of the commitment period (e.g., a firm can receive payment for a service only when it is completed).[7] Or it can be the present value of flow profits (e.g., profits from launching a new product once its development is complete), and flow costs that are incurred for the duration of the project's commitment.

[6] Capacity constraints could arise due to a limited number of skilled scientists and engineers, limited physical space to run experiments, etc. Capacity constraints rather than budget constraints are used also in the literature on rational inattention, see Sims (2010).

[7] The expected value is $v = E_t\{\delta^{s-t}\widetilde{v}\}$ where $s - t$ is the time to completion and $\widetilde{v}$ the value obtained at the end of the period.

## 2.1 A Decision Maker's Project Selection

Denote by $V_0$ the value function when the decision maker is not committed, before realization of the project's return, and denote by $V_1$ the value function for the committed decision maker. When the decision maker is committed, he cannot select another project. Thus,

$$V_1 = \delta \left[ pV_0 + (1-p) V_1 \right]. \tag{1}$$

When he is free, he chooses to select or not to select so as to maximize:

$$\max \left\{ \underbrace{v + \delta \left[ pV_0 + (1-p) V_1 \right]}_{\text{payoff if select}}, \underbrace{\delta V_0}_{\text{payoff if reject}} \right\}.$$

Note that the continuation value if the firm selects is equal to $V_1$. Thus, a project is selected if $v \geq v_0$ where:

$$v_0 = (\delta V_0 - V_1). \tag{2}$$

The value in the state without commitment is:

$$V_0 = \int_{\underline{v}}^{v_0} \delta V_0 f(v) dv + \int_{v_0}^{\overline{v}} (v + V_1) f(v) dv. \tag{3}$$

If projects do not require commitment ($p = 1$), the selection threshold is $v_0 = 0$. But when projects require commitment $(p < 1)$, some positive return projects are rejected, $v_0 > 0$.

**Proposition 1** *(i) There exists a unique solution to the system (1)-(3), with a threshold value for project selection $v_0 \in (0, \overline{v})$. (ii) The threshold $v_0$ is higher when the commitment required for each project is expected to last longer (lower $p$); and when the decision maker is more patient (higher $\delta$). (iii) The threshold $v_0$ is at least as high for a distribution of returns that either first order stochastically dominates another or is a mean-preserving spread of another.*

By Proposition 1, the return from a selected project is expected to be higher in an industry in which firms typically commit to projects that take a long time to complete (low $p$ in our model). In the pharmaceutical industry, for example, it takes about ten years to bring a drug to the market (see Nicholas, 1994). The threshold for selection is higher for a dominating distribution because there is a higher probability that a better project will arise in the following periods. The threshold for selection is higher for the spread because the firm can enjoy the higher return project while rejecting those with lower return.

## 2.2 Two-Project Capacity Constraint

We now consider a decision maker who can work on two projects at a time. We compare project selection of this less constrained firm with the single project capacity firm in Section 2.1. We use the two-project capacity model later, when comparing a duopoly with a joint venture.

We denote the value functions of the decision maker with a two-project capacity with $W_i$, to distinguish from that of the single project capacity values $V_i$. The index $i \in \{0, 1, 2\}$ in $W_i$ refers to the number of projects to which the decision maker is committed. In state 2, the decision maker cannot select a project. The optimal choices in states 0 and 1 are given by thresholds of selection $w_0$ and $w_1$, respectively. The value in state 2 (when the firm is committed to two projects) is:

$$W_2 = \delta \left[ p^2 W_0 + 2p(1-p) W_1 + (1-p)^2 W_2 \right]. \tag{4}$$

In state 1, the threshold level satisfies:

$$w_1 = \delta (pW_0 + (1-p) W_1) - W_2. \tag{5}$$

The value function in state 1 is:

$$W_1 = \int_{w_1}^{\overline{v}} (v + W_2) f(v) \, dv + F(w_1) \delta (pW_0 + (1-p) W_1). \tag{6}$$

In state 0, the threshold level satisfies:

$$w_0 = \delta (1-p)(W_0 - W_1). \tag{7}$$

The value function in state 0 is:

$$W_0 = \int_{w_0}^{\overline{v}} [v + \delta (pW_0 + (1-p) W_1)] f(v) \, dv + F(w_0) \delta W_0. \tag{8}$$

Equations (4)-(8) define the solution to the optimal decision of the firm who can work on at most two projects at a time. In a model without commitment ($p = 1$), the selection thresholds would be $w_1 = w_0 = 0$. When $p < 1$, selecting projects requires commitment. Committing to a project is more costly to the firm when it has already committed to one project before, and its threshold would be higher. Moreover, comparing firms with different capacity constraints, we find that the single project decision maker's threshold is higher than both thresholds of selection of the two-project capacity firm. It is even higher than the threshold the two-project capacity firm uses when it is already committed to one project. Intuitively, for the two-project capacity firm, when all its resources are committed, the expected time until at least one of the two commitments is relieved is shorter than for the committed single project capacity firm.

**Proposition 2** *Suppose $p < 1$. For a decision maker who can work on at most two projects, the selection threshold is higher when one project is underway than when it is not committed, and these thresholds are lower than the selection threshold of the single project capacity firm: $v_0 \geq w_1 > w_0 > 0$. The first inequality is strict if $p > 0$.*

An implication of Proposition 2 is that a firm that has a more stringent capacity constraint selects less projects and that the projects it selects are of higher average return, compared to what the less constrained firm does. It also spends a higher fraction of its time waiting for a project to select. The constrained firm does not select some of the low return projects that the less constrained firm would have selected.

# 3 Strategic Project Selection

## 3.1 The Game

We consider competition between two firms $A$ and $B$. Every period, a new project opportunity with a value $v$ drawn from the distribution $F(v)$ arises. If one firm is committed to an earlier project and the other is free, the free firm can decide whether to select the project. If both firms are free, one is chosen at random (with a probability $\frac{1}{2}$) to be that period's leader- or to be the first firm to make a decision to select the project or not. If the period's leader selects the project, the follower cannot take on a project in that period. If the period's leader rejects the project (which is an irreversible decision), the follower can decide whether to select it. Thus, when both firms are free, they play sequentially.[8] If neither firm selected the project, it disappears and a new project opportunity arises in the next period.[9] As we assumed in Section 2.1, a firm can work on at most one project at a time and the commitment of resources ends each period with a probability $p$.

Our solution concept is a Markov perfect equilibrium which rules out non-credible threats, and restricts to strategies that depend only on "payoff-relevant" history (see Maskin and Tirole, 1988)[10]. The Markov perfect equilibrium is a standard solution concept for dynamic games. A state in our game is characterized by $(i, j)$ which describe commitment: $i = 0$ and $i = 1$ indicate whether the focal firm (the firm currently deciding, or whose value is being defined) is free or committed respectively, and $j = 0$ or $j = 1$ indicate if the other firm is free or committed. Additionally, $k$ indicates the identity of the focal firm ($k = A$ or $B$) and $l$ (which is only meaningful in state $(0, 0)$) indicates whether the firm is a leader ($l = 1$) or the follower ($l = 2$). For simplicity, we will refer to $(i, j)$ as the state omitting the identity of the firm when this is clear from the context.

We use dynamic programming to find an equilibrium. The value function (capturing the present discounted value of profits before realization of the period's project's return) in any state is denoted by $V^k_{i,j}$. We focus on characterizing symmetric equilibria. Symmetric equilibria are simpler. We show in Proposition 3 that a pure strategy symmetric equilibrium exists. When a symmetric equilibrium exists, it seems to be a natural solution concept for a symmetric game. We briefly discuss asymmetric equilibria at the end of Section 3.2, and provide more details in the online appendix. In this section we focus on symmetric

[8] We briefly discuss a simultaneous version of the game in the conclusions and provide more details in an online appendix.

[9] In "real life," a firm might be able to reconsider a project that it had rejected in an earlier period. In the context of our model, we could think of the project arriving again at a later date. For tractability, however, we abstract from the effect that rejected projects might have on the distribution of quality of future projects.

[10] Maskin and Tirole (1988) introduce a class of alternating move infinite horizon models of duopoly competition in which firms make short-run commitments to a strategy (quantity or price) for a finite period of time.

equilibria, thus, $V^A_{i,j} = V^B_{i,j} = V_{i,j}$. Symmetric equilibrium strategies are characterized by thresholds $\left(v_{0,1}, v^1_{0,0}, v^2_{0,0}\right)$, where $v_{0,1}$ is the threshold for the non-committed firm in a state $(0,1)$, $v^1_{0,0}$ and $v^2_{0,0}$ are the thresholds for the period's leader (the first mover) and follower. We say that a threshold is interior if it is in the range $(\underline{v}, \overline{v})$, so that the lowest value project is rejected, while the highest value project is accepted.

## 3.2 Analysis

We derive the conditions that define the equilibrium thresholds and the values. The value in state $(1,1)$ is given by:

$$V_{1,1} = \delta\left[p^2 V_{0,0} + p(1-p)V_{1,0} + p(1-p)V_{0,1} + (1-p)^2 V_{1,1}\right]. \tag{9}$$

In state $(0,1)$, the free firm chooses to select or to reject the project so as to maximize:

$$\max\left\{\underbrace{v + \delta\left[p^2 V_{0,0} + p(1-p)V_{1,0} + p(1-p)V_{0,1} + (1-p)^2 V_{1,1}\right]}_{\text{select}}, \underbrace{\delta\left(pV_{0,0} + (1-p)V_{0,1}\right)}_{\text{reject}}\right\}.$$

Note that the continuation value when the firm selects a project is equal to $V_{1,1}$. Thus, an interior threshold return for selection satisfies:

$$v_{0,1} = \delta\left(pV_{0,0} + (1-p)V_{0,1}\right) - V_{1,1}. \tag{10}$$

Using the threshold $v_{0,1}$, the value functions in states $(0,1)$ and $(1,0)$ are:

$$V_{0,1} = \int_{v_{0,1}}^{\overline{v}} (v + V_{1,1}) f(v)\, dv + F(v_{0,1})\, \delta\left(pV_{0,0} + (1-p)V_{0,1}\right), \tag{11}$$

$$V_{1,0} = \int_{v_{0,1}}^{\overline{v}} V_{1,1} f(v)\, dv + F(v_{0,1})\, \delta\left(pV_{0,0} + (1-p)V_{1,0}\right). \tag{12}$$

In state $(0,0)$, one of the firms is chosen at random to be the leader who can consider the project first. If this firm does not select the project, then the follower faces the choice:

$$\max\left\{\underbrace{v + \delta\left(pV_{0,0} + (1-p)V_{1,0}\right)}_{\text{select}}, \underbrace{\delta V_{0,0}}_{\text{reject}}\right\}. \tag{13}$$

If the threshold is interior, it satisfies:

$$v_{0,0}^2 = \delta (1-p)(V_{0,0} - V_{1,0}). \tag{14}$$

Returning to the leader's decision, if $v \geq v_{0,0}^2$, so that the second firm will select if it has the opportunity to do so, then the first firm faces the choice:

$$\max \left\{ \underbrace{v + \delta (pV_{0,0} + (1-p) V_{1,0})}_{\text{select}}, \underbrace{\delta (pV_{0,0} + (1-p) V_{0,1})}_{\text{reject} \Rightarrow \text{rival selects}} \right\}. \tag{15}$$

The firm would select this project if $v \geq \widetilde{v}$ where:

$$\widetilde{v} = \delta (1-p)(V_{0,1} - V_{1,0}). \tag{16}$$

If $v < v_{0,0}^2$, so that the follower will not select the project even if the leader rejected it, then the leader faces essentially the same choice as that of the follower after the leader rejected a project. Thus, for $v < v_{0,0}^2$, neither firm selects. Combining the results, the leader selects if $v \geq v_{0,0}^1$ where:

$$v_{0,0}^1 = \max \left\{ v_{0,0}^2, \widetilde{v} \right\} \geq v_{0,0}^2. \tag{17}$$

If $\widetilde{v} > v_{0,0}^2$, then there is a range of project returns $v_{0,0}^2 \leq v < v_{0,0}^1$ so that the leader rejects, but the follower selects. The equation that defines the value function in state $(0,0)$ is given by:

$$V_{0,0} = \frac{1}{2} \int_{v_{0,0}^2}^{\overline{v}} v f(v)\, dv + F\left(v_{0,0}^2\right) \delta V_{0,0} + \left[1 - F\left(v_{0,0}^2\right)\right] \delta \left[ pV_{0,0} + (1-p) \frac{V_{0,1} + V_{1,0}}{2} \right]. \tag{18}$$

**Proposition 3** *(i) For the game described in this section, there exists a pure strategy Markov perfect equilibrium with interior thresholds* $\left(v_{0,1}, v_{0,0}^1, v_{0,0}^2\right) \in (\underline{v}, \overline{v})^3$.

*(ii) For long duration projects* $(p = 0)$, *the symmetric equilibrium is unique.*

Existence of a symmetric pure strategy equilibrium follows from Brouwer's fixed point theorem. Given our assumptions on the parameters of the model, in any symmetric pure strategy equilibrium, all the thresholds must be interior. That is, when a firm is free to select, it does not reject all projects nor does it accept all projects. Uniqueness is not always guaranteed, but we can show that the equilibrium is unique at least for certain parameter values including the case $p = 0$, where a firm can only select one project (an assumption often made in literature on irreversible investment). When $p = 0$, the system of equilibrium equations becomes simpler as a committed firm expects no additional payoffs $V_{1,1} = V_{1,0} = 0$, and the free firm in state $(0,1)$ faces the same problem as that of a single decision maker, therefore $v_{0,1} = \widetilde{v} = v_0$ which we have shown to be unique. We are only left with finding one threshold, $v_{0,0}^2$, which we verify must be unique.

For $p = 0$, the observations we made above on the values and thresholds when at least one firm is committed are true even if we allow for asymmetric equilibria. Thus, the only potential source of asymmetry in the equilibrium strategies could be in the thresholds $v_{0,0}^{k,2}$. With an additional assumption on the distribution function ($f'(x) \geq 0$), there is no asymmetric equilibrium.[11]

## 3.3 Strategic Rejection of Projects

When no commitment of resources is needed $(p = 1)$, the equilibrium thresholds are $v_{0,1} = v_{0,0}^1 = v_{0,0}^2 = 0$. In this case, a follower will never select a project that the leader rejected. For $p < 1$, we show that $v_{0,0}^2 < v_{0,0}^1$ so that there is a range of intermediate value projects, $v \in \left(v_{0,0}^2, v_{0,0}^1\right)$, that the leader rejects and the follower selects. This result might seem initially surprising as one might expect the leader to select all the "good enough" projects leaving only projects that the follower would not want either. To understand why the leader's threshold is higher, consider the trade-off between selecting and rejecting in state $(0,0)$ for the follower, given in (13) and for the leader given in (15). The payoff from selecting the project would be same for the leader and follower, $v + \delta\left(pV_{0,0} + (1-p)V_{1,0}\right)$. For the follower, rejecting would result in a continuation value $\delta V_{0,0}$ since no firm would select a project this period. But for the leader, rejecting a project that the follower would select provides a higher continuation value $\delta\left(pV_{0,0} + (1-p)V_{0,1}\right)$ since with probability $(1-p)$ the leader would have

[11] More details about the derivations of asymmetric equilibrium conditions are available in an online appendix. In an asymmetric equilibrium (when one exists), if $v_{0,0}^{B,2} < v_{0,0}^{A,2}$, when firm $A$ is the leader, firm $A$ has a similar selection pattern as in the symmetric game, in particular it necessarily rejects some projects that the follower would adopt, firm $B$ will accept some projects that firm $A$ would have rejected.

a busy rival in the following period, guaranteeing that the leader could select the project in the next period, if a high value project should arise.

**Proposition 4** *For $p < 1$, in a symmetric Markov perfect equilibrium, the thresholds for selection in state $(0,0)$ satisfy $v^1_{0,0} > v^2_{0,0}$; in the range $v^1_{0,0} > v \geq v^2_{0,0}$, the leader rejects the project but the follower selects it. Additionally, $v^1_{0,0} \geq v_{0,1}$ (with a strict inequality for $p \in (0,1)$), so that a firm rejects more projects when its rival is free.*

Next, we compare the equilibrium with selection patterns in two benchmarks. First, in Section 3.4, we compare the behavior of a firm with a one-project capacity constraint with that of an equally constrained monopolist (a single project decision maker). Then, in Section 3.5, we compare the outcome of the non-cooperative game with that of two firms that maximize joint payoffs.

## 3.4 Competition and Project Selection

A vast literature in economics debates the relation between market structure and innovation (see Gilbert, 2006, for a survey). We examine the effects of competition on project selection in the context of our model. We compare the selection strategies in the duopoly game (Section 3.1) with the selection behavior of the decision maker (Section 2.1). We show that the threshold return for a project to be selected is lower when two firms compete than when there is a single decision maker who is constrained to select one project. Thus, (assuming project opportunities arise at the same rate regardless of market structure) more projects are selected in a duopoly market than in a monopoly market. However, since the threshold of selection is higher for the monopolist, the monopolist will tend to work on projects that have a higher expected return.

**Proposition 5** *For $p \in (0,1)$, selection thresholds are lower in duopoly competition than for a monopolist: $v^1_{0,0} < v_0$.*

The comparison in Proposition 5 is based on the assumption that the project return is drawn from the same distribution in the single decision maker's problem as in the game. If the firm enjoys a higher return from any project when it is alone in the market, the distribution of returns in the decision maker's problem may dominate that in the duopoly case. As we have shown in Proposition 1, this would imply an even higher threshold of selection. So the result of Proposition 4 holds, even if the monopolist earns more from each project compared with the duopolist.

### 3.5 Joint Decision versus the Non-cooperative Game

The two-project capacity version we analyzed in Section 2.2 allows us to compare project selection decisions of strategic non-cooperative firms with the decision of a joint venture. We are interested in two comparisons. First, we compare the selection thresholds when one firm is committed and one is free, that is, we compare the threshold of the joint venture that is committed to one project but can still select another $(w_1)$, to the selection threshold of the non-cooperative firm that is not committed and has a rival who is committed $(v_{0,1})$. Second, we compare the threshold of the joint venture when it is free of commitments $(w_0)$, with the selection threshold above which at least one of the competing firms selects the project when they are both free $\left(v_{0,0}^2\right)$.

**Proposition 6** *For $p \in (0,1)$, a non-cooperatively competing duopolist has higher selection thresholds compared with the jointly optimal decision maker, $w_1 < v_{0,1}$ and $w_0 < v_{0,0}^2$.*

To prove this proposition, we first argue that the joint decision maker can obtain at least as high a value in state 0 as the sum of values of both firms in the game in state $(0,0)$, i.e., $W_0 \geq 2V_{0,0}$. This is true because the joint decision maker can mimic the equilibrium selection strategies in the game. We then derive the given inequalities on thresholds from the system of dynamic programming equations. Proposition 6 suggests that competition between firms with capacity constraints on project selection results in these firms setting too high a bar for selection and thus rejecting too many projects compared with what would be optimal for them in a joint decision.

We compare our results in the case $p = 0$ with those in Weeds (2002). In Weeds (2002), when a symmetric equilibrium arises (which holds only for some range of parameters), then strategic interaction increases the time to first investment compared with a joint venture. This result is similar to our finding in Proposition 6, which shows that the thresholds are lower in the joint optimal strategy. As Weeds notes, "this is in stark contrast to the usual presumption that the fear of preemption speeds up investment." In Weeds' model, the threshold for the second investment in the joint optimal strategy is higher than that in the non-cooperative symmetric equilibrium, while in our model, the threshold for the second investment is lower. The intuition for delay in first investment in Weeds (2002) is that strategic firms fear setting off a patent race. In our model, the joint venture rejects less projects because of its greater flexibility in using capacity, and because when committed to two projects, the joint venture expects to be freed from at least one commitment sooner than a committed non-cooperative firm. In Weeds' model, sometimes only an asymmetric equilibrium exists and when this is the case, the joint venture would delay investment, which is in contrast to our result.

# 4 Extensions

We consider a number of possible generalizations of our model.

## 4.1 Oligopoly

Consider an oligopoly with $n$ firms. Let us indicate states by $(i, j)$ where $i \in \{0, 1\}$ indicates if the focal firm is free or busy, and $j \in \{0, 1, .., n-1\}$ is the total number of other firms (not including the focal firm) that are committed at the time the firm is deciding on a project. We assume only long duration projects in this sub-section, i.e., $p = 0$.[12]

For committed firms in any state $(1, j)$ the values are:

$$V_{1,j} = 0 \text{ for } j = 0, ..., n-1.$$

In state $(0, n-1)$, when all other firms are committed, the free firm faces the same problem as the decision maker in Section 2.1. The threshold and value are:

$$v_{0,n-1} = \delta V_{0,n-1} \text{ and } V_{0,n-1} = \int_{v_{0,n-1}}^{\overline{v}} v f(v)\, dv + F(v_{0,n-1})\, \delta V_{0,n-1}.$$

In state $(0, j)$, $j$ firms are busy and $n-j$ firms are free. The project is considered sequentially by the free firms in a random order, until one selects it or all reject it. If no other firm selected

[12] We analyze the more general case with $p \in [0, 1]$ in the online appendix for $n = 3$ firms.

the project earlier, the threshold for the last firm to consider the project, the $(n-j)$th free firm, is:

$$v_{0,j}^{n-j} = \delta V_{0,j}.$$

We argue that all the other firms in the sequence of decision makers that consider the project earlier have the same threshold, which is higher than that of the last firm. The first firm to consider the project rejects some projects that the last firm selects. If $v < v_{0,j}^{n-j}$, then the last firm will reject the project. Knowing this, the second to last mover, the $(n-j-1)$th firm, also faces the choice between $v$ if it selects and $\delta V_{0,j}$ if it rejects. Thus, the second to last firm will also reject. If $v \geq v_{0,j}^{n-j}$, the last free firm will select the project if it will be rejected by all other firms. The second to last firm has the choice $\max\{v, \delta V_{0,j+1}\}$. Thus, its threshold is:

$$v_{0,j}^{n-j-1} = \max\left\{\delta V_{0,j+1}, v_{0,j}^{n-j}\right\} = \max\left\{\delta V_{0,j+1}, \delta V_{0,j}\right\}.$$

Going to the $(n-j-2)$th mover, if $v < v_{0,j}^{n-j}$, this firm knows that no one will accept it, and it also rejects. If $v \geq v_{0,j}^{n-j}$, if the $(n-j-2)$th mover rejects, one of the other two firms will accept the project. The $(n-j-2)$th mover thus solves $\max\{v, \delta V_{0,j+1}\}$ which is the same as the problem that the $(n-j-1)$th firm had, and thus their thresholds are equal: $v_{0,j}^{n-j-1} = v_{0,j}^{n-j-2}$. A similar argument works for every remaining leading firm. Hence, when $j$ firms are committed, the last mover's threshold is $v_{0,j}^{n-j} = \delta V_{0,j}$, and for all but the last mover, the thresholds are equal: $v_{0,j}^{1} = ... = v_{0,j}^{n-j-1} = \max\{\delta V_{0,j+1}, \delta V_{0,j}\} \geq v_{0,j}^{n-j} = \delta V_{0,j}$. The value in state $(0,j)$ for $j = 0, ..., n-1$ is:

$$V_{0,j} = \frac{1}{n-j}\int_{v_{0,j}^{n-j}}^{\overline{v}} vf(v)\,dv + \left(1 - F\left(v_{0,j}^{n-j}\right)\right)\left(1 - \frac{1}{n-j}\right)\delta V_{0,j+1} + F\left(v_{0,j}^{n-j}\right)\delta V_{0,j}.$$

**Proposition 7** *When $p = 0$, there exists a unique symmetric equilibrium to the oligopoly game. In equilibrium, the value of a free firm is larger the more rival firms are busy, $V_{0,j+1} > V_{0,j}$. Selection thresholds satisfy: $v_{0,j}^{1} = ... = v_{0,j}^{n-j-1} > v_{0,j}^{n-j}$, for $j = 0, ..., n-1$.*[13]

Thus, as in the duopoly case, also in the oligopoly model, the firms that move early reject some projects that the last mover accepts. However, as the number of free firms $n-j$

[13] The proof of this proposition is provided in the online appendix.

approaches infinity, almost all the projects are either selected by the first-moving firm or not selected at all. The proof follows from the derivations above, and uses induction on the number of firms.

## 4.2 A Dominant Firm

Asymmetry between firms could be another reason why firms sometimes reject projects that their rivals then select. Obviously, if the leader values a project less than the follower, the follower might select a project that the leader rejected. Even if two firms agree on the value of a project, one firm might be more likely to evaluate projects before the other when both are free. For example, one firm may be better known, or easier to approach, or project ideas may start within this firm.

We consider here the extreme case where one firm is always the first to consider projects (and there are no payoff externalities). The dominant firm, firm $A$, has no benefit from having a busy rival, as it is always first to choose. The dominant firm maximizes payoffs by ignoring its opponent, and thus using the threshold that maximizes the decision maker's payoffs (derived in Section 2.1) whenever it is free. Firm $B$, the follower, will best respond to this strategy. Firm $B$'s values and thresholds in a best response to the leader's strategy solve the system (9)-(18) that we derived in Section 3.2 for the symmetric game, only with minor modifications: we use the leader's threshold as $v_0$ in equation (12) defining $V_{1,0}$ (now $V_{1,0}^B$) and we replace (18), the value in state $(0,0)$, with the following:

$$V_{0,0}^B = (1 - F(v_0))\,\delta\left(pV_{0,0}^B + (1-p)\,V_{0,1}^B\right) + \int_{v_{0,0}^{B,2}}^{v_0} \left(v + V_{1,1}^B\right) f(v)\,dv + F\left(v_{0,0}^{B,2}\right)\delta V_{0,0}^B. \quad (19)$$

**Proposition 8** *For $p < 1$, in a Markov perfect equilibrium, in state $(0,0)$, the leader selects projects of returns higher than $v_0$ (the unique solution to the system (1)-(3) in Section 2.1) and rejects projects of lower value. There exists a threshold $v_{0,0}^{B,2} < v_0$ so that in state $(0,0)$, in the range $v_0 > v \geq v_{0,0}^{B,2}$, the dominant firm rejects the project, but the follower selects it.*

In the symmetric model, we showed that the benefit from having a busy rival provides an incentive for the leader to reject projects that the follower would accept. In the perfect dominance case described in this subsection, the dominant firm does not benefit from a busy rival, but the follower still selects some projects that the leader rejects because he faces an inferior selection of projects. More generally, if firm $A$ has an advantage in the form of a probability $\beta \in \left(\frac{1}{2}, 1\right)$ to get priority, both motives can play a role in creating a wedge between the leader's and the follower's thresholds.

## 4.3 Payoff Externalities

So far we assumed that a firm that selects a project obtains the payoff $v$ and its opponent does not obtain an immediate payoff. We now address the possibility of a payoff externality. We assume that for a project with return $v$ to the selecting firm, the return to the other firm is $\gamma v$, where $\gamma \in (-1, 1)$, i.e., the externality is smaller in magnitude than the return for the selecting firm. Negative externalities ($\gamma < 0$) may exist, for example, if the firms compete in the marketplace and the project gives the selecting firm a cost advantage. Positive externalities ($\gamma > 0$) may exist, for example, when there are technology spillovers. Payoffs $v$ and $\gamma v$ can also represent Stackelberg payoffs with the selecting firm becoming the market leader in a new product market. The analysis of the model with externalities follows in the same way as that in Section 3. The system of equilibrium equations remains the same except for the following changes. Accounting for payoff externalities, in state $(1, 0)$ the value function which was given in (12) becomes:

$$V_{1,0} = \int_{v_{0,1}}^{\overline{v}} (\gamma v + V_{1,1}) f(v) \, dv + F(v_{0,1}) \, \delta \left( p V_{0,0} + (1-p) V_{1,0} \right). \tag{20}$$

In state $(0, 0)$, the leader's threshold $\widetilde{v}$ in (16) for accepting projects that the follower would select is given by:

$$\widetilde{v} = \frac{\delta (1-p)}{(1-\gamma)} (V_{0,1} - V_{1,0}),$$

and the value function in state $(0, 0)$ becomes:

$$V_{0,0} = \frac{1+\gamma}{2} \int_{v_{0,0}^2}^{\overline{v}} v f(v) \, dv + \delta F\left(v_{0,0}^2\right) V_{0,0} + \delta \left[1 - F\left(v_{0,0}^2\right)\right] \left[ p V_{0,0} + (1-p) \frac{V_{0,1} + V_{1,0}}{2} \right].$$

The results of Proposition 4 continue to hold. Clearly, however, the equilibrium thresholds depend on the externality $\gamma$. On the one hand, in state $(0, 0)$, for project values that would be selected by the follower if rejected by the leader, a larger $\gamma$ increases the immediate value from rejecting, $\gamma v$. This suggests a positive effect on $v_{0,0}^1$. On the other hand, a larger $\gamma$ reduces the incentive to having a busy rival and provides a force that reduces thresholds through its effect on future values. From (11) and (20), we have:

$$v_{0,0}^1 = \underbrace{\frac{1}{1-\gamma}}_{\text{immediate effect}\uparrow} \underbrace{\delta(1-p)(V_{0,1}-V_{1,0})}_{\text{future value effect}\downarrow} = \frac{\delta(1-p)}{(1-\gamma)} \frac{(1-\gamma)\int_{v_{0,1}}^{\overline{v}} vf(v)\,dv}{[1-F(v_{0,1})\delta(1-p)]} = \frac{\delta(1-p)\int_{v_{0,1}}^{\overline{v}} vf(v)\,dv}{[1-F(v_{0,1})\delta(1-p)]}.$$

When $p=0$, the threshold $v_{0,1}$ is constant with respect to $\gamma$ (since the rival will never be able to select another project), the two effects on $v_{0,0}^1$ exactly cancel out so that $v_{0,0}^1$ remains constant as well. But the threshold $v_{0,0}^2$ declines. When $p>0$ but still small, we show that all thresholds decline.

**Proposition 9** *There exists $0<\overline{p}\leq 1$ so that the equilibrium thresholds $\left(v_{0,1}, v_{0,0}^2, \widetilde{v}\right)$ are continuously differentiable in $p$ and in $\gamma$ for all $(p,\gamma)\in R=[0,\overline{p})\times(-1,1)$. In $R$, $\frac{dv_{0,0}^2}{d\gamma}<0$; also, $\frac{d\widetilde{v}}{d\gamma}\leq 0$ and $\frac{dv_{0,1}}{d\gamma}\leq 0$ with a strict inequality for $p\in(0,\overline{p})$.*

Proposition 9 implies that (for $(p,\gamma)$ in the range $R$) in the presence of positive externalities, the thresholds of selection are lower (more projects are selected) than without externalities. Intuitively, with positive externalities, firms benefit less from having a committed rival, and would be less concerned about committing their own resources.

Lastly, we comment on the polar cases $|\gamma|=1$. If $\gamma=(-1)$, we have a zero sum game, $V_{0,0}=V_{1,1}=0$, $V_{0,1}=-V_{1,0}$, and all the thresholds are equal: $v_{0,0}^1=\widetilde{v}=v_{0,0}^2=v_{0,1}$. If $\gamma=1$, $V_{0,1}=V_{1,0}$. In state $(0,0)$, whenever the follower would accept a project $\left(v>v_{0,0}^2\right)$, the leader is indifferent between accepting and rejecting. There are many (payoff equivalent) equilibria. In one of them, the inequality $v_{0,0}^1=v_{0,0}^2$ holds.

# 5 Concluding Remarks

Projects often require firms to commit limited resources, preventing them from selecting other projects while they are committed. Since more promising projects could arise during the time a firm is committed to a project it selected earlier, constrained firms reject some profitable projects. In a strategic environment, project selection by one firm can change the profitability of a rival, as well as the rival's opportunity to take on projects. In the symmetric game, we show that a firm sometimes rejects a project that will then be selected by a rival, as this can lessen future competition on projects. In an asymmetric game, the follower may accept lower value projects than the leader also because it faces an inferior distribution of projects.

We consider the relation between market structure and innovations, in an environment with perpetual selection of projects. We show that a duopolist has lower selection thresholds than an identical firm who operates as a monopoly. Thus, competition induces more projects to be selected, but the average quality of projects selected by the monopolist is higher. If however the two firms are able to jointly make selection decisions, then the selection thresholds of the joint decision maker are lower than in the non-cooperative equilibrium. The joint decision maker can better manage capacity.

In attempting to maintain tractability, we made certain simplifying assumptions. In our model, if the firm is committed, it cannot select another project until the commitment ends. In reality, it is likely that firms can sometimes be released from a commitment at some cost. In our model, the cost of abandoning a project is high so that the commitment is binding. If the cost is not too high, then when a new project of high enough return arises, the firm might find it worthwhile to abandon an old project. We expect the results to be qualitatively similar, with perhaps lower selection thresholds because the commitment is less binding.[14] An additional interesting but challenging direction for future work is allowing for the possibility that the duration of a project ($p$) is correlated with the size of the prize ($v$). If high prizes are associated with long commitments, it might not be possible to characterize strategies as thresholds of selection. Formal analyses of these extensions are left for future work.

In analyzing strategic interactions, our model assumes sequential decisions. We argue

[14] When there is a cost to abandon the project, and project returns are only obtained conditional on the project being completed, the problem becomes much more complex because in addition to the thresholds of a free firm, when committed, the firm would need to decide whether to abandon the project. This decision could depend on the value of the project it is currently committed to (and possibly that of its rival).

that in many economic environments this assumption is reasonable, e.g., clients likely approach service providers sequentially. However, there may be some markets in which firms simultaneously decide on project selection, or where the order of sequential moves might be endogenous. If, in each period, firms simultaneously decide on selection, and each gets the project with equal probability when they both attempt to select, there is a range of intermediate value project returns for which firms randomize the decision to select.

Our analysis is focused on the strategic behavior of the firms that select projects. If, however, project opportunities arise when independent innovators propose them, they might also act strategically so as to extract surplus from the selecting firms. The game played in each period might take the form of an auction or a bargaining game. These extensions are interesting directions for future work.

# A Appendix: Proofs

**Proof of Proposition 1.** (i) Re-arranging (1) and (3), then substituting into (2), we obtain the following implicit definition of the selection decision threshold $v_0$:

$$\frac{1-\delta\left(1-p\right)}{\delta\left(1-p\right)}v_0=\int_{v_0}^{\overline{v}}\left(v-v_0\right)f(v)dv. \tag{21}$$

From $(21)$, we know that $v_0$ is the solution to $g(x)=0$ where $g(.)$ takes the following form:

$$g(x)=\frac{\left[1-\delta\left(1-p\right)\right]}{\delta\left(1-p\right)}x-\int_{x}^{\overline{v}}\left(v-x\right)f(v)dv. \tag{22}$$

Evaluating $g(x)$ at $x=0$ and at $x=\overline{v}$, we find that $g(0)<0$ and $g(\overline{v})>0$. Additionally,

$$g'\left(x\right)=\frac{\left[1-\delta\left(1-p\right)\right]}{\delta\left(1-p\right)}+\left[1-F\left(x\right)\right]>0.$$

This implies that for given $\delta$ and $p$, there exists a unique solution to (22) in the range $(0,\overline{v})$.

(ii) Consider $g(v_0;\delta,p)$ referring to $g$ as defined in (22) evaluated at $v_0$ and accounting $\delta,p$ as arguments of the function. Implicit differentiation of $g(v_0;\delta,p)=0$ shows that:

$$\begin{aligned}\frac{\partial g}{\partial v_0}\frac{dv_0}{dp}+\frac{\partial g}{\partial p} &= 0\Rightarrow\frac{dv_0}{dp}=-\frac{\partial g}{\partial p}/\frac{\partial g}{\partial v_0},\\ \frac{\partial g}{\partial v_0}\frac{dv_0}{d\delta}+\frac{\partial g}{\partial \delta} &= 0\Rightarrow\frac{dv_0}{d\delta}=-\frac{\partial g}{\partial \delta}/\frac{\partial g}{\partial v_0}.\end{aligned}$$

From the proof of (i), we know that $\frac{\partial g}{\partial v_0} > 0$. Hence,

$$sign\left(\frac{dv_0}{dp}\right) = sign\left(-\frac{\partial g}{\partial p}\right) = sign\left[-\frac{v_0}{\delta(1-p)^2}\right] < 0.$$

Similarly,

$$sign\left(\frac{dv_0}{d\delta}\right) = sign\left(-\frac{\partial g}{\partial \delta}\right) = sign\left[\frac{v_0}{\delta^2(1-p)}\right] > 0.$$

(iii) Consider two cumulative distribution functions $F^a$ and $F^b$, defined on the interval $[\underline{v}, \overline{v}]$ so that $F^b$ first order stochastically dominates $F^a$, i.e., $F^b(v) \leq F^a(v)$. For each distribution, the threshold return $v_0^i$ solves $g^i(v) = 0$ where $g^i$ is as defined in (22) with the distribution $F^i$, $i = a, b$. Using integration by parts, we rearrange the function $g^i$ and write it as:

$$g^i(v_0) = \frac{[1-\delta(1-p)]}{\delta(1-p)}v_0 - \int_{v_0}^{\overline{v}} \left[1 - F^i(v)\right] dv, \; i = a, b. \tag{23}$$

Therefore, $g^b(v) \leq g^a(v)$. In particular, $g^b(v_0^a) \leq g^a(v_0^a) = 0$. Since the function $g^b$ is increasing, this implies that $v_0^a \leq v_0^b$, which is the desired conclusion.

Consider two cumulative distribution functions $F^b$ and $F^a$ defined on $[\underline{v}^b, \overline{v}^b]$ and $[\underline{v}^a, \overline{v}^a] \subset [\underline{v}^b, \overline{v}^b]$ respectively so that $F^b$ is a mean-preserving spread of $F^a$, (or $F^b$ second order stochastically dominates (SOSD) $F^a$). Define $F^a(v) = 0$ for all $v < \underline{v}^a$, and $F^a(v) = 1$ for all $v > \overline{v}^a$. Then,

$$g^b(v_0^a) = g^b(v_0^a) - g^a(v_0^a) = \int_{v_0^a}^{\overline{v}^b} \left[F^b(v) - F^a(v)\right] dv \underset{\text{by SOSD}}{\leq} 0.$$

Because the function $g^b$ is increasing, $g^b(v_0^a) \leq 0$ implies that $v_0^a \leq v_0^b$, which is the desired conclusion. ■

**Proof of Proposition 2.** Using (5), we can re-write (6) as follows:

$$W_1 = \int_{w_1}^{\overline{v}} (v - w_1) f(v) dv + \delta(pW_0 + (1-p)W_1). \tag{24}$$

Similarly, using (7), we can re-write (8) as follows:

$$W_0 = \int_{w_0}^{\overline{v}} (v - w_0) f(v) dv + \delta W_0. \tag{25}$$

Substituting the two equations above into (7) and re-arranging, we get:

$$\frac{[1-\delta(1-p)]}{\delta(1-p)}w_0 = \int_{w_0}^{\overline{v}} (v-w_0) f(v)\, dv - \int_{w_1}^{\overline{v}} (v-w_1) f(v)\, dv. \tag{26}$$

Again, substituting $(4), (7)$ and $(24)$ into $(5)$, we get:

$$\begin{aligned} w_1 &= \delta(pW_0 + (1-p)W_1) - \frac{\delta}{\left[1-\delta(1-p)^2\right]}\left[p^2 W_0 + 2p(1-p)W_1\right] \\ &= \frac{\delta(1-p)^2}{\left[1-\delta(1-p)^2\right]}(1-\delta)W_0 - \left[1 - \frac{2p}{\left[1-\delta(1-p)^2\right]}\right] w_0 \\ &= \frac{\delta(1-p)^2}{\left[1-\delta(1-p)^2\right]}\int_{w_0}^{\overline{v}} (v-w_0) f(v)\, dv - \left[1 - \frac{2p}{\left[1-\delta(1-p)^2\right]}\right] w_0. \end{aligned}$$

Substituting (26) into this expression and re-arranging, we get:

$$\frac{[1-\delta(1-p)]}{\delta(1-p)}w_1 = \int_{w_1}^{\overline{v}} (v-w_1) f(v)\, dv + \frac{p}{\delta(1-p)^2}(w_0 - w_1). \tag{27}$$

We re-write (26) and (27) substituting the function $g(.)$ that was defined in $(22)$:

$$g(w_0) = -\int_{w_1}^{\overline{v}} (v-w_1) f(v)\, dv, \tag{28}$$

$$g(w_1) = \frac{p}{\delta(1-p)^2}(w_0 - w_1). \tag{29}$$

For all $p < 1$, if $w_0 \geq w_1$, then $g(w_1) \geq 0 > g(w_0)$ which implies $w_1 > w_0$ (because $g(.)$ is increasing), a contradiction! Hence, $w_1 > w_0$ for all $p < 1$. Finally, $w_0 > 0$ follows from (26) and $w_1 > w_0$.

Note that we assume project values are drawn from the same distribution in either the two projects or the single project capacity model. From the proof of Proposition 1(i), we know that $v_0$ is the solution to a strictly increasing function $g(v_0) = 0$. From above, we know that $w_1 > w_0$ for all $p < 1$ and by (29), we know that $g(w_1) < 0$. It follows that:

$$g(v_0) = 0 > g(w_1) > g(w_0).$$

This implies that $v_0 > w_1 > w_0$ because $g(.)$ is increasing. ■

**Proof of Proposition 3.** An equilibrium is defined by the system (9)-(18). Accounting for corner solutions, we re-write the thresholds as:

$$
\begin{aligned}
v_{0,1} &= median\left\{\underline{v}, \delta\left(pV_{0,0}+(1-p)V_{0,1}\right)-V_{1,1}, \overline{v}\right\}, \\
v_{0,0}^{2} &= median\left\{\underline{v}, \delta(1-p)\left(V_{0,0}-V_{1,0}\right), \overline{v}\right\}, \\
\widetilde{v} &= median\left\{\underline{v}, \delta(1-p)\left(V_{0,1}-V_{1,0}\right), \overline{v}\right\}, \\
v_{0,0}^{1} &= \max\left\{v_{0,0}^{2}, \widetilde{v}\right\}.
\end{aligned}
$$

These thresholds are in the range $[\underline{v}, \overline{v}]$, and written as a function of the four values $\{V_{1,1}, V_{0,1}, V_{1,0}, V_{0,0}\}$. Thus the system of equations that defines the equilibrium can be reduced to a system of 4 equations in these 4 unknowns. We substitute (9) into (11) and (12) to obtain this system in values $\{V_{1,1}, V_{0,1}, V_{1,0}, V_{0,0}\}$:

$$
\begin{aligned}
V_{1,1} &= \delta\left[p^2V_{0,0}+p(1-p)V_{1,0}+p(1-p)V_{0,1}+(1-p)^2V_{1,1}\right], \\
V_{0,1} &= \left[\begin{array}{c} \int_{v_{0,1}}^{\overline{v}}\left(v+\delta\left[p^2V_{0,0}+p(1-p)V_{1,0}+p(1-p)V_{0,1}+(1-p)^2V_{1,1}\right]\right)f(v)\,dv \\ +F\left(v_{0,1}\right)\delta\left(pV_{0,0}+(1-p)V_{0,1}\right) \end{array}\right], \\
V_{1,0} &= \left[\begin{array}{c} \int_{v_{0,1}}^{\overline{v}}\delta\left[p^2V_{0,0}+p(1-p)V_{1,0}+p(1-p)V_{0,1}+(1-p)^2V_{1,1}\right]f(v)\,dv \\ +F\left(v_{0,1}\right)\delta\left(pV_{0,0}+(1-p)V_{1,0}\right) \end{array}\right], \\
V_{0,0} &= \int_{v_{0,0}^{2}}^{\overline{v}}\frac{1}{2}vf(v)\,dv+\delta F\left(v_{0,0}^{2}\right)V_{0,0}+\delta\left[1-F\left(v_{0,0}^{2}\right)\right]\frac{(1-p)\left(V_{0,1}+V_{1,0}\right)+2pV_{0,0}}{2},
\end{aligned}
$$

where the thresholds are given above. The right hand side is a continuous function from the compact set $\left[\frac{1}{1-\delta}\underline{v}, \frac{1}{1-\delta}\overline{v}\right]^4$ to the same set. By Brouwer's theorem, a fixed point for this system exists. The fixed point satisfies the system (9)-(18) and it is therefore an equilibrium.

To show that the equilibrium must be interior given our assumptions, we consider separately each possible case in which a threshold either equals $\underline{v}$ or $\overline{v}$. The derivations are simple, but lengthy, and we defer them to an online appendix. Lastly, to show that for $p=0$ the symmetric equilibrium is unique, we note that in this case $V_{1,1}=V_{1,0}=0$. The equations defining $V_{0,1}$ and $v_{0,1}$ become identical to those defining the value and threshold in the decision maker case. As shown in the proof of Proposition 1, there is a unique threshold

$v_{0,1} = v_0$. From (16), we find that $\widetilde{v} = v_{0,1}$. Lastly, it remains to show that the threshold $v_{0,0}^2$ is unique given all the other thresholds are unique, which we can verify by substituting $v_{0,0}^2 = \delta V_{0,0}$ into the expression for $V_{0,0}$ in (18) and rearranging to obtain that $v_{0,0}^2$ is the unique solution to:

$$\left(\frac{1-\delta}{\delta}\right) v_{0,0}^2 = \frac{1}{2}\int_{v_{0,0}^2}^{\overline{v}} v f(v)\, dv + \left[1 - F\left(v_{0,0}^2\right)\right] \frac{(1-p)\, v_{0,1} - 2(1-p)\, v_{0,0}^2}{2}.$$

■

**Proof of Proposition 4.** By (17), $v_{0,0}^1 = \max\left\{v_{0,0}^2, \widetilde{v}\right\} \geq v_{0,0}^2$, and $v_{0,0}^1 > v_{0,0}^2$ iff $\widetilde{v} > v_{0,0}^2$. We need to show that $\widetilde{v} > v_{0,0}^2$. We first reduce the system (9)-(18) that defines the equilibrium to a system of three equations in the three unknowns: $\widetilde{v}, v_{0,1}$ and $v_{0,0}^2$, using substitution and simple manipulations. Details of the derivations of this system are available from the authors upon request. The reduced system is given by:

$$\frac{[1-\delta(1-p)]}{\delta(1-p)}\widetilde{v} - \int_{v_{0,1}}^{\overline{v}} (v - \widetilde{v}) f(v)\, dv = 0, \tag{30}$$

$$\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,1} - \int_{v_{0,1}}^{\overline{v}} (v - v_{0,1}) f(v)\, dv - \frac{p\left(v_{0,0}^2 - v_{0,1}\right)}{\delta(1-p)^2} = 0, \tag{31}$$

$$\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,0}^2 - \left(\begin{array}{c} \int_{v_{0,0}^2}^{\overline{v}} \left(v - v_{0,0}^2\right) f(v)\, dv - \frac{1}{2}\int_{v_{0,0}^2}^{\overline{v}} (v - \widetilde{v}) f(v)\, dv \\ -\int_{v_{0,1}}^{\overline{v}} (v - v_{0,1}) f(v)\, dv + \int_{v_{0,1}}^{\overline{v}} (v - \widetilde{v}) f(v)\, dv \end{array}\right) = 0. \tag{32}$$

It is easy to verify that $v_{0,0}^2 = v_{0,1} = \widetilde{v}$ will be a solution to the system iff $p = 1$. Now suppose $p < 1$.

By (30) and (31),

$$v_{0,1} \geq \widetilde{v} \Leftrightarrow \frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,1} - \int_{v_{0,1}}^{\overline{v}} (v - v_{0,1}) f(v)\, dv \geq 0 \Leftrightarrow p\left(v_{0,0}^2 - v_{0,1}\right) \geq 0. \tag{33}$$

Thus, for $p = 0$, we have $v_{0,1} = \widetilde{v}$ and for $p > 0$ we have either $v_{0,0}^2 \geq v_{0,1} \geq \widetilde{v}$ or $\widetilde{v} \geq v_{0,1} \geq v_{0,0}^2$.

By the definition of $g$ in (22) and from (31) and (32), we have:

$$
\begin{aligned}
& g\left(v_{0,0}^2\right)-g\left(v_{0,1}\right) \\
= & \left(\begin{array}{c}
\left[\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,0}^2-\int\limits_{v_{0,0}^2}^{\overline{v}}\left(v-v_{0,0}^2\right) f(v)\, dv\right] \\
-\left[\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,1}-\int\limits_{v_{0,1}}^{\overline{v}}\left(v-v_{0,1}\right) f(v)\, dv\right]
\end{array}\right) \\
= & \left[\begin{array}{c}
-\frac{1}{2} \int\limits_{v_{0,0}^2}^{\overline{v}}(v-\widetilde{v}) f(v)\, dv-\int\limits_{v_{0,1}}^{\overline{v}}\left(v-v_{0,1}\right) f(v)\, dv+\int\limits_{v_{0,1}}^{\overline{v}}(v-\widetilde{v}) f(v)\, dv \\
-\left[\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,1}-\int\limits_{v_{0,1}}^{\overline{v}}\left(v-v_{0,1}\right) f(v)\, dv\right]
\end{array}\right]
\end{aligned}
$$

$$
\begin{aligned}
= & \left(-\frac{1}{2} \int\limits_{v_{0,0}^2}^{\overline{v}}(v-\widetilde{v}) f(v)\, dv+\int\limits_{v_{0,1}}^{\overline{v}}(v-\widetilde{v}) f(v)\, dv-\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,1}\right) \\
= & \left(-\frac{1}{2} \int\limits_{v_{0,0}^2}^{\overline{v}}(v-\widetilde{v}) f(v)\, dv+\frac{[1-\delta(1-p)]}{\delta(1-p)} \widetilde{v}-\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,1}\right), \text{ from (30)} \\
= & -\frac{1}{2} \int\limits_{v_{0,0}^2}^{\overline{v}}(v-\widetilde{v}) f(v)\, dv-\frac{[1-\delta(1-p)]}{\delta(1-p)}\left(v_{0,1}-\widetilde{v}\right).
\end{aligned}
$$

If $v_{0,0}^2 \geq v_{0,1} \geq \widetilde{v}$, then:

$$
\int\limits_{v_{0,0}^2}^{\overline{v}}(v-\widetilde{v}) f(v)\, dv \leq \int\limits_{v_{0,1}}^{\overline{v}}(v-\widetilde{v}) f(v)\, dv .
$$

Also,

$$
\left[\left(-\frac{1}{2}\right) \int\limits_{v_{0,0}^2}^{\overline{v}}(v-\widetilde{v}) f(v)\, dv\right]<0 .
$$

Hence, if $v_{0,0}^2 \geq v_{0,1} \geq \widetilde{v}$, then $\left[g\left(v_{0,0}^2\right)-g\left(v_{0,1}\right)\right]<0$ which is a contradiction because $g$ is strictly increasing. Therefore, by (33), if $p=0$, it must be that $\widetilde{v}=v_{0,1}>v_{0,0}^2$, and if $p>0$, then it has to be the case that $\widetilde{v}>v_{0,1}>v_{0,0}^2$. ■

**Proof of Proposition 5.** From the proof of Proposition 1, we know that $v_0$ is the solution to $g(v_0)=0$, where $g(.)$ is an increasing function defined in (22).

From the proof of Proposition 4, we know that $v_{0,0}^{1} = \widetilde{v} > v_{0,1}$ for all $p \in (0,1)$ and hence we can rewrite (30) as follows:

$$\begin{aligned} \frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,0}^{1} &= \int_{v_{0,1}}^{\overline{v}} \left(v - v_{0,0}^{1}\right) f(v)\, dv \\ \text{or, } g\left(v_{0,0}^{1}\right) &= -\int_{v_{0,1}}^{v_{0,0}^{1}} \left(v_{0,0}^{1} - v\right) f(v)\, dv < 0 = g(v_0)\,. \end{aligned}$$

This implies that $v_{0,0}^{1} < v_0$. ■

**Proof of Proposition 6.** We use the system of equations for the two-project capacity version in Section 2.2. We follow three steps to prove this proposition.

Step 1: We argue that the optimal project selection strategy of the two-project capacity firm yields a value that is at least as large as the sum of equilibrium values of the two firms in the game, i.e., $W_0 \geq 2V_{0,0}$. The reason why this is true is that the two-project capacity decision maker can mimic the selection behavior of the strategic firms by setting thresholds:

$$w_0 = v_{0,0}^{2} \text{ and } w_1 = v_{0,1}.$$

With these thresholds, a project is implemented by the decision maker if and only if one of the firms would have selected the project in the game. Using the thresholds from the game, the values become:

$$W_1 = (V_{0,1} + V_{1,0})\,,\ W_0 = 2V_{0,0},\ \text{and } W_2 = 2V_{1,1}.$$

These values satisfy equations (4), (6) and (8) in the two-project capacity problem. Thus, the two-project capacity decision maker can achieve at least $2V_{0,0}$ in state 0, and hence it must be that in the optimal strategy, $W_0 \geq 2V_{0,0}$.

Step 2: We have shown in Proposition 4 that $v_{0,0}^{2} < v_{0,0}^{1} = \widetilde{v}$. This implies that:

$$\int_{v_{0,0}^{2}}^{\overline{v}} \left(v - v_{0,0}^{2}\right) f(v)\, dv > \int_{v_{0,0}^{2}}^{\overline{v}} (v - \widetilde{v})\, f(v)\, dv.$$

We subtract $\delta V_{0,0}$ from both sides of (18), rearrange and substitute (14) and (16) to find that:

$$(1-\delta) V_{0,0} = \int_{v_{0,0}^{2}}^{\overline{v}} \left(v - v_{0,0}^{2}\right) f(v)\, dv - \frac{1}{2} \int_{v_{0,0}^{2}}^{\overline{v}} (v - \widetilde{v})\, f(v)\, dv, \tag{34}$$

and hence we have:

$$\begin{aligned}(1-\delta)2V_{0,0} &= 2\int_{v_{0,0}^2}^{\overline{v}} \left(v-v_{0,0}^2\right) f(v)\,dv - \int_{v_{0,0}^2}^{\overline{v}} (v-\widetilde{v})\, f(v)\,dv \\ &> 2\int_{v_{0,0}^2}^{\overline{v}} \left(v-v_{0,0}^2\right) f(v)\,dv - \int_{v_{0,0}^2}^{\overline{v}} \left(v-v_{0,0}^2\right) f(v)\,dv \\ &= \int_{v_{0,0}^2}^{\overline{v}} \left(v-v_{0,0}^2\right) f(v)\,dv.\end{aligned}$$

Suppose, by contradiction, that $w_0 \geq v_{0,0}^2$. Then,

$$(1-\delta)2V_{0,0} > \int_{v_{0,0}^2}^{\overline{v}} \left(v-v_{0,0}^2\right) f(v)\,dv \geq \int_{w_0}^{\overline{v}} (v-w_0)\, f(v)\,dv = (1-\delta)\,W_0.$$

Where the last equality is derived from (8) and (7). But we argued that $W_0 \geq 2V_{0,0}$, a contradiction! Hence, it must be that $w_0 < v_{0,0}^2$.

Step 3: We argue that because $v_{0,0}^2 > w_0$, it must follow that $v_{0,1} > w_1$. By contradiction, if it was the case that $v_{0,1} \leq w_1$, then using the definition of $g(.)$ in (22) in the proof of Proposition 1, we have:

$$\begin{aligned} g(v_{0,1}) &\leq g(w_1) \text{ because } g(.) \text{ is increasing} \\ &\Rightarrow \frac{p\left(v_{0,0}^2 - v_{0,1}\right)}{\delta(1-p)^2} \leq \frac{p(w_0-w_1)}{\delta(1-p)^2} \text{ by (31) and (29)} \\ &\Rightarrow \left(v_{0,0}^2 - v_{0,1}\right) \leq (w_0 - w_1) \\ &\Rightarrow \left(v_{0,0}^2 - w_0\right) \leq (v_{0,1} - w_1) \leq 0 \\ &\Rightarrow v_{0,0}^2 \leq w_0, \text{ a contradiction!}\end{aligned}$$

Hence, it must be that $v_{0,1} > w_1$. ■

**Proof of Proposition 8.** For the leader (firm $A$), the strategy $v_{0,1}^A = v_{0,0}^A = v_0$, which is the optimal strategy for the decision maker, maximizes payoffs. The values for the leader are then:

$$V_{1,1}^A = V_{1,0}^A = V_1 \text{ and } V_{0,1}^A = V_{0,0}^A = V_0.$$

For firm $B$, the best response to the leader's strategy is derived from the same system of equations as in Section 3.2, using $v_{0,1}^{A} = v_0$ for the value in state $(1,0)$ and (19) for the value in state $(0,0)$. We would like to show that $v_{0,0}^{B,2} < v_0$. Suppose by contradiction that $v_{0,0}^{B,2} \geq v_0$. Then by (19), we get:

$$V_{0,0}^{B} = (1 - F(v_0))\,\delta\left(pV_{0,0}^{B} + (1-p)\,V_{0,1}^{B}\right) + F(v_0)\,\delta V_{0,0}^{B}.$$

Reducing the system of equations to one with only thresholds (as we did in the proof of Proposition 4), we obtain:

$$\begin{aligned}
\frac{[1-\delta(1-p)]}{\delta(1-p)} v_0 &= \int_{v_0}^{\overline{v}} (v - v_0)\, f(v)\, dv, \\
\frac{[1-\delta(1-p)]}{\delta(1-p)} \widetilde{v}^{B} &= \int_{v_0}^{\overline{v}} \left(v - \widetilde{v}^{B}\right) f(v)\, dv + \int_{v_0}^{v_{0,1}^{B}} \left(v_{0,1}^{B} - v\right) f(v)\, dv, \\
\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,1}^{B} &= \int_{v_{0,1}^{B}}^{\overline{v}} \left(v - v_{0,1}^{B}\right) f(v)\, dv + \frac{p\left(v_{0,0}^{B,2} - v_{0,1}^{B}\right)}{\delta(1-p)^2}, \\
\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,0}^{B,2} &= \int_{v_0}^{\overline{v}} \left(v - v_{0,0}^{B,2}\right) f(v)\, dv - \int_{v_0}^{\overline{v}} \left(v - v_{0,1}^{B}\right) f(v)\, dv.
\end{aligned}$$

If $p = 0$, we immediately obtain from the first three equations that $v_{0,1}^{B} = v_0 = \widetilde{v}^{B}$, and the fourth implies $v_{0,0}^{B,2} < v_0$.

Consider $p \in (0,1)$. Define $g_1(x) = \frac{[1-\delta(1-p)]}{\delta(1-p)} x - \int_{v_0}^{\overline{v}} (v - x)\, f(v)\, dv$. Note that $g_1(.)$ is strictly increasing. Hence,

$$\begin{aligned}
v_{0,0}^{B,2} &\geq v_0 \Leftrightarrow g_1\left(v_{0,0}^{B,2}\right) \geq g_1(v_0) = 0 \\
&\Leftrightarrow \int_{v_0}^{\overline{v}} \left(v - v_{0,1}^{B}\right) f(v)\, dv \leq 0 \Rightarrow v_{0,1}^{B} > v_0 \Leftrightarrow g\left(v_{0,1}^{B}\right) > g(v_0) = 0 \Leftrightarrow v_{0,0}^{B,2} > v_{0,1}^{B}.
\end{aligned}$$

Hence,

$$v_{0,0}^{B,2} > v_{0,1}^{B} > v_0.$$

The equation for $v_{0,0}^{B,2}$ implies:

$$\frac{[1-\delta(1-p)]}{\delta(1-p)}v_{0,0}^{B,2} = \left[\begin{array}{l} \int\limits_{v_{0,0}^{B,2}}^{\overline{v}} \left(v - v_{0,0}^{B,2}\right) f(v)\,dv - \int\limits_{v_0}^{v_{0,0}^{B,2}} \left(v_{0,0}^{B,2} - v\right) f(v)\,dv \\ -\int\limits_{v_{0,1}^{B}}^{\overline{v}} \left(v - v_{0,1}^{B}\right) f(v)\,dv + \int\limits_{v_0}^{v_{0,1}^{B}} \left(v_{0,1}^{B} - v\right) f(v)\,dv \end{array}\right]$$

$$\text{or, } g\left(v_{0,0}^{B,2}\right) = -\left(\int\limits_{v_0}^{v_{0,0}^{B,2}} \left(v_{0,0}^{B,2} - v\right) f(v)\,dv - \int\limits_{v_0}^{v_{0,1}^{B}} \left(v_{0,1}^{B} - v\right) f(v)\,dv\right) - \int\limits_{v_{0,1}^{B}}^{\overline{v}} \left(v - v_{0,1}^{B}\right) f(v)\,dv.$$

Now $\int\limits_{v_0}^{x} (x-v) f(v)\,dv$ is increasing in $x > v_0$, hence $\int\limits_{v_0}^{v_{0,0}^{B,2}} \left(v_{0,0}^{B,2} - v\right) f(v)\,dv > \int\limits_{v_0}^{v_{0,1}^{B}} \left(v_{0,1}^{B} - v\right) f(v)\,dv \Rightarrow$ $g\left(v_{0,0}^{B,2}\right) < 0 = g(v_0) \Leftrightarrow v_{0,0}^{B,2} < v_0$, a contradiction! ■

**Proof of Proposition 9.** The equilibrium thresholds in the model with externalities is a solution to a $H\left(v_{0,1}, v_{0,0}^2, \widetilde{v}; p, \gamma\right) = 0$, where $h_1$ is given by $(30)$, $h_2$ is given by (31) and generalizing (32) to account for the externality,

$$h_3 = \frac{[1-\delta(1-p)]}{\delta(1-p)}v_{0,0}^2 - \left(\begin{array}{l} \int\limits_{v_{0,0}^2}^{\overline{v}} \left(v - v_{0,0}^2\right) f(v)\,dv - \frac{(1-\gamma)}{2} \int\limits_{v_{0,0}^2}^{\overline{v}} (v - \widetilde{v}) f(v)\,dv \\ -\int\limits_{v_{0,1}}^{\overline{v}} (v - v_{0,1}) f(v)\,dv + (1-\gamma) \int\limits_{v_{0,1}}^{\overline{v}} (v - \widetilde{v}) f(v)\,dv \end{array}\right). \quad (35)$$

This system is continuously differentiable, and at $p = 0$, the determinant of its Jacobian matrix is strictly positive. By the implicit function theorem, around $p = 0$, the thresholds are continuously differentiable. Thus, there is a non-empty set $R = [0, \overline{p}) \times (-1, 1)$ where the thresholds are continuously differentiable. When $p = 0$, $v_{0,1} = \widetilde{v} = v_0$, are constant with respect to $\gamma$. Implicitly differentiating $h_3 = 0$, we find that $\frac{dv_{0,0}^2}{d\gamma} < 0$. For $p > 0$ in $R$, (30) implies that $sign\left(\frac{d\widetilde{v}}{d\gamma}\right) = sign\left(\frac{dv_{0,1}}{d\gamma}\right)$ and (31) implies that $sign\left(\frac{dv_{0,1}}{d\gamma}\right) = sign\left(\frac{dv_{0,0}^2}{d\gamma}\right)$.

Suppose by contradiction that there exists $(p, \gamma) \in R$ so that $\frac{dv_{0,0}^2}{d\gamma} > 0$. Since we have proved that for any $\gamma$ and $p = 0$, this threshold is decreasing in $\gamma$, there must exist $p^* \in (0, p)$ for which $\frac{dv_{0,0}^2}{d\gamma} = 0$. For this $p^*$, by equation $(31)$, $\frac{dv_{0,1}}{d\gamma} = 0$ and then by (30) also $\frac{d\widetilde{v}}{d\gamma} = 0$. But this cannot be since then by (35), we would need to have $\frac{\partial h_3}{\partial \gamma} = 0$ which would imply that:

$$\int\limits_{v_{0,1}}^{\overline{v}} (v - \widetilde{v}) f(v)\,dv - \frac{1}{2} \int\limits_{v_{0,0}^2}^{\overline{v}} (v - \widetilde{v}) f(v)\,dv = 0. \quad (36)$$

Substituting (36) into (35), we would obtain:

$$\frac{[1-\delta(1-p)]}{\delta(1-p)}v_{0,0}^2 - \int_{v_{0,0}^2}^{\overline{v}} \left(v - v_{0,0}^2\right) f(v)\, dv = \left(-\int_{v_{0,1}}^{\overline{v}} (v - v_{0,1}) f(v)\, dv\right) < 0.$$

Re-arranging (36), we have:

$$\frac{1}{2}\int_{v_{0,0}^2}^{v_{0,1}} (v - \widetilde{v}) f(v)\, dv = \frac{1}{2}\int_{v_{0,1}}^{\overline{v}} (v - \widetilde{v}) f(v)\, dv.$$

We know that $\int_{v_{0,1}}^{\overline{v}} (v - \widetilde{v}) f(v)\, dv > 0$ by (30) and $\widetilde{v} > 0$. Thus, $\int_{v_{0,0}^2}^{v_{0,1}} (v - \widetilde{v}) f(v)\, dv > 0$. But by Proposition 4, $\widetilde{v} > v_{0,1} > v_{0,0}^2$ so this implies $\int_{v_{0,0}^2}^{v_{0,1}} (v - \widetilde{v}) f(v)\, dv < 0$, a contradiction! Thus, $v_{0,0}^2$ decreases with $\gamma$, and so do the other two thresholds which we have shown move in the same direction as $v_{0,0}^2$. ■

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