

DEFENSIVE PUBLICATIONS IN AN R&D RACE

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This paper considers a multistage dynamic R&D race in which the competitors strategically publish research findings. Publications change the prior art, thus affecting patentability. Firms publish when they are behind in the race and their rival is close to winning it. Publication sets back both competitors and gives the follower a chance to catch up. Publications prolong the race. Firms are more likely to publish the more patient they are, and the higher their probability of success. Asymmetry between the firms generates additional incentives to publish such as protecting profits from a previous patent and increasing a strong firm's probability of winning. When firms face a joint decision on publications and the intensity of research, publications substitute investment.

1. INTRODUCTION

Intellectual property, particularly patents, is of vital and growing importance to the global economy. Although patent races have received considerable attention in the economic literature, the strategic use of publications in innovation races has not. This paper focuses on one important reason for public disclosure of research results: defensive publication. Publishing details of their research findings, firms establish new prior art and thus strategically affect the patentability of related innovations. Firms can use publications to defend their own inventions from being patented by another company and to increase their probability of leapfrogging the leader and winning the race.

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Typically, profit-maximizing firms either seek patent protection for their innovations or keep them secret from rivals. Detailed technical disclosures may reveal information to rivals without providing the publishing firm intellectual property rights. Interestingly we find that firms' strategic behavior involves publications that disclose research findings. The main novelty of this paper is in identifying a role for publications in a dynamic R&D model. Importantly, and in contrast to most previous economic models of R&D races, in this paper, the state of the prior art is endogenous.¹ A firm's progress is compared not only to its rival's progress but also to the state of the prior art.

Legally, to be entitled for patent protection, claims must define a useful, novel, nonobvious, and enabled invention. An invention must sufficiently improve upon the prior state of the art. Patent examiners review and analyze patent applications in conjunction with the state of the prior art. According to the *Manual of Patent Examining Procedures*: "By far the most frequent ground of rejection is on the ground of unpatentability in view of the prior art."² Detailed publications regarding an innovation become part of the prior art and affect the patentability of related innovation. As a recent *New York Times* article points out, "Such disclosure, known as defensive publishing, is an increasingly common tactic for protecting intellectual property."³

As is evident from the existence of commercial services for defensive publications,⁴ firms use defensive publishing as part of their business strategy. Many leading companies publish research disclosures.⁵ Baker and Mezzetti (2005) in a case study of IBM's disclosures, find that approximately one out of six patents issued to IBM cite as prior art at least one article from the IBM Technical Disclosure Bulletin and most of these patents cite articles published less than 5 years earlier. They interpret this finding as evidence that IBM is often disclosing information about patents that it is actively pursuing.⁶ Note that during this same time (1996–2001) more than 75% of the patents that cited

1. Exceptions include Parchomovsky (2000), Lichtman et al. (2000), and Baker and Mezzetti (2005). See Section 6.

2. See section 706.02 in the *Manual of Patent Examining Procedure* (1993).

3. See Milstein (2002).

4. The following services provide clear, dated publications that are easily available to patent examiners: (i) *Research Disclosure*: "a defensive-type publication . . . available to companies who . . . seek a low cost alternative, or supplement, to obtaining patents." (ii) *IP.com, Inc.*: a web based service "IP.com offers a defensive publication service that enables inventors to rapidly place inventions into the public domain." (iii) Large corporations' bulletins or disclosure journals. (iv) U.S. Patent and Trademark office: statutory invention registration (SIR).

5. "90% of the world's leading companies have published disclosures in *RD*." See <http://www.researchdisclosure.com>.

6. They acknowledge that this conclusion is not straightforward, but their findings are consistent with it.

IBM's Bulletin were patents assigned to other companies (e.g. Apple Computers, Compaq Computer Corporation, Intel, Hewlett-Packard, Sun Microsystems, Texas Instruments) indicating that these disclosures were also relevant to their competitors' research.⁷

Following Baker and Mezzetti's (2005) insight, that prior art references to research disclosures in an issued patent may point to a race with strategic disclosure, we looked for a specific example involving competing innovating firms. Note that if defensive publications are successful, we shouldn't expect to observe preempted patents but rather delayed patents. In 1992 AGFA published in the journal *Research Disclosure* a disclosure titled: "A Lithographic Printing Plate."⁸ In 1997 Kodak filed for a utility patent titled "Direct write lithographic printing plates," which provides a prior art reference to AGFA's disclosure.⁹ In 1995, AGFA itself also filed for a related patent citing the same disclosure.¹⁰ The model provides a possible explanation for AGFA's publication. The companies engaged in closely related research. If AGFA suspected that Kodak may be close to patenting the technology, publishing could have forestalled it and thus was worthwhile even though AGFA intended to continue pursuing a patent.¹¹

According to Eisenberg (2000) who studied public disclosure of genomic information,

It might even be possible for a firm that is lagging in a race to forestall the patent claims of a swifter rival through publications that enrich the prior art enough to limit what may be patented in the future. This possibility might explain why it is often the laggards rather than the leaders in DNA sequencing races that sing the praise of the public domain.¹² (p. 72)

7. These are results of a search in Micropatent's data base: www.micropat.com.

8. *Research Disclosure* Jan. 1992, #33303, A Lithographic Printing Plate by Vermeersch of Agfa-Gevaert N.V.

9. US Patent number 5,962,188. Granted in 1999.

10. See US patent 5607810 "Method for making a lithographic printing plate requiring no wet processing." Granted in 1997. There is no earlier patent assigned to AGFA referencing this disclosure.

11. Reality is more complex than the model. It is likely that AGFA did not know for sure if Kodak would attempt to patent knowledge that their disclosure could preempt, but assigning a large enough probability that this could happen suffices for the intuitive argument. Also note that while the model has a single winner, in reality both firms may obtain related patents.

12. Eisenberg acknowledged that this issue is not free of doubt. For example, publication of ESTs most likely would not have made the corresponding full-length gene obvious. Moreover, such disclosure may have been patentable. Nevertheless, for the intuition in this paper to apply the important point is that the "prize" from the innovation was to be had only after more significant progress was made and that publication may have prolonged the race

This paper models the use of defensive publications in an R&D race. In particular it explains why followers (laggards) publish. In the model two firms are engaged in a multistage patent race. To win the race, a firm needs to be the first to accomplish n discrete steps above the prevailing prior art. The winner of the race enjoys a prize while the loser obtains no payoff but incurs the loss of its R&D investment. Publication alters the state of the prior art so that more innovation steps are required. It sets back both competitors relative to their goal of advancing n steps above the state of the prior art. In other words, the race that we consider has no fixed finish line. Firms' progress and the state of the prior art are observed. There is uncertainty with respect to R&D, which is modeled as a Poisson process. To emphasize the defensive purpose of research disclosure, publications are assumed to have no direct payoff, and firms do not possess market power over an innovation once it is made public.

We first consider publications in a simple model in which the instantaneous probability of success is fixed. Here, the only decision the firms face is whether or not to publish. In a symmetric race, in a Markov perfect equilibrium a competitor who is behind in the race publishes research results in order to set back the leader when the leader is very close to obtaining a patent. Because publishing prolongs the race and delays the prize, the follower will want to wait for as long as possible before publishing. By doing so the follower avoids unnecessary publications in the event he makes favorable progress in R&D. For the same reason, the follower will choose to publish the least possible to prevent the competitor from winning, and will do so only when he is far enough behind. Patient firms publish whenever they fall behind and their rival has completed $n - 1$ steps.

If the firms are not identical, additional incentives to publish arise. A firm that holds a patent over a core technology may find it worthwhile to publish in order to enjoy the stream of payoffs from the patent it holds for a longer time. When the firms bear different costs of investment, the low-cost firm may gain from publishing if by prolonging the race it drives the high-cost firm out of the race. When firms have different probabilities of success, the firm with the advantage would find that publishing increases its probability of winning the race.

We also investigate the use of publications when the probability of success increases with R&D effort. Here we consider a race with a convex cost function. Having the ability to respond to a change in their competitive positions with a change in investment, firms may avoid publications. Publications sometimes serve as a substitute for R&D investment. Whether or not firms publish in equilibrium depends on the cost function and the patentability requirement. When the cost of

increasing the probability of success is high, we find again that followers publish defensively.

The paper proceeds as follows: Section 2 sets up the model. Section 3 describes defensive publications in a symmetric race. Section 4 shows incentives to publish in asymmetric races. Section 5 considers the race with investment. Section 6 is on related literature. Section 7 concludes.

2. THE MODEL

2.1 PAYOFFS AND STATES

Two identical firms, a and b , are engaged in a patent race.¹³ The race is a multistage race; an innovation is composed of discrete steps. Time is continuous and the horizon is infinite. Innovation occurs according to a Poisson process. The date at which a new step will be discovered is random and depends on both firms' instantaneous probabilities of success (the hazard rates λ^i).¹⁴ Each firm's hazard rate increases with its research effort. Research has a flow cost $c(\lambda^i)$.

An innovation is evaluated against the state of the prior art which is a measure of the current knowledge in the market regarding the innovation. The prior art is normalized to zero at the beginning of the race. To win the race,¹⁵ a firm needs to be the first to accomplish a fixed number of innovation steps, n , above the state of the prior art. The prior art and the level of progress made by each firm are common knowledge.¹⁶ However, firms cannot observe the content of their rival's research, unless the rival disclosed that information. A firm may choose to publish research results. Publication changes the state of the prior art.

13. Patent races are central to industrial organization theory. The applicability of the race analogy is sometimes questioned. This is a highly simplified view of the world, but it helps understand strategic R&D considerations. Recent R&D races that caught the attention of the media include the race to patent the HIV/Aids virus, the race to patent the SARS virus, and the human genome race.

14. In a Poisson ("Memoryless") R&D process, at each state s and moment t , the probability that to date $t + \tau$ none of the firms advanced a step is $e^{-(\lambda_s^a + \lambda_s^b)\tau}$. Firm i is the first to progress the next step with probability $\lambda_s^i d\tau$. See Tirole (2001).

15. In a first-to-invent patent regime (as in United States) we can think of the winner of an R&D race as being the first to conceive a patentable innovation in a first to file the winner is the first to file for the patent.

16. Common knowledge of the prior art is reasonable because (1) previous patents are public, (2) publishing firms want to be sure the patent examiner has access to their ideas, (3) innovating firms make efforts to be informed about the prior art because it can help their research and in order to avoid legal costs. In practice, there may be doubt about whether the patent office is informed of the prior art. Common knowledge of each other's progress may result from the firms' need to attract venture capital, from interactions between employees or other spillover. Imperfect information, although more realistic, is much more complex to analyze.

We assume there is a very small cost for publishing.¹⁷ The publication fee is fixed for each step published and is independent of the state. For simplicity, we derive the limiting case in which this cost is zero unless we note otherwise. A small cost will play a role in eliminating multiplicity of equilibria. Each successful step that was published increases the prior state of the art by one step. A simultaneous publication of one step by both firms has the same effect as a publication by one of the firms, since it is assumed to add the same information to the state of the prior art.

We define a set of state variables. Each state $\mathbf{s} = (s^a, s^b)$ denotes the situation in which firm $i = a$ or b had s^i unpublished innovation steps above the state of the prior art. Let the state space be

$$S = \{(s^a, s^b) : s^i = 0, 1, 2, \dots, n\}.$$

The number s^i increases by 1 whenever firm i experiences a success. If firm i publishes the results of k steps, the state of the prior art increases by k steps. Consequently, the number of steps above the prior art for each firm decreases by k successes (or is equal to zero if a firm had fewer than k successes). This assumption regarding the effect of publications on the state of the world implicitly embeds an assumption that any published step raises the bar for both firms. A simple case in which this could hold is if both firms choose the same path of innovation.¹⁸

The value of the patent is v for the winning firm (which accomplished $n = s^i > s^j$ steps) and it is zero for the loser. The tie-breaking rule is for each firm to obtain the patent with probability $\frac{1}{2}$, and get the value $\frac{v}{2}$. This, however, is a zero-probability event. The flow cost of research is $-(\lambda)dt$. The payoff of the game at state \mathbf{s} is simply the discounted expected value of rewards for firm i , that is, expected gain from the patent minus the expected cost. This expected payoff depends on the firms' strategies.

2.2 STRATEGIES AND EQUILIBRIUM

A strategy may depend at every point of time on the entire history of the game up to that point. Markov strategies are such that past play influences current play only through its effect on the state variables. In every state $\mathbf{s} = (s^a, s^b)$, $s^i < n$ firms make their publication choice,

17. Posting a document on IP.com's web site costs only \$155 and requires no lawyers. See Milstein (2002).

18. In reality, often firms innovate in different paths. So in some cases publications would not affect the patentability of rival's inventions. Suppose the parties know the state (s^a, s^b) , but that they may choose different paths of innovation. If there is a probability $0 < \pi < 1$ that the firms choose a similar path so that publications would be effective, the incentive to disclose is lower than with $\pi = 1$, but if π is large enough, qualitatively we can still expect most of the results to hold.

followed by a choice of effort when no publication occurred. Instead, if a firm published, there is a transition to a new state. When $s^i = n$, a terminal node was reached, firm j can no longer publish and delay firm i 's victory. In the publication stage, let the set of actions for firm i be $A^i(\mathbf{s}) = \{0, 1, \dots, s^i\}$. In the investment stage, let the set of actions be R_+ , all possible hazard rates. A pure Markov strategy for player i is a function:

$$f^i : S \rightarrow A^i(\mathbf{s}) \times R_+.$$

A partially mixed equilibrium strategy allows the firm to mix over its choice of publications. As we will see later, we can restrict attention to the choice of publishing one step or not publishing at all. Define the partially mixed strategy as a function

$$\sigma^i : S \rightarrow [0, 1] \times R_+,$$

where the first component of this function is the probability of publications and the second is the choice of flow investment.

A pure Markov perfect equilibrium (MPE) is a pair of pure Markov strategies that yield a Nash equilibrium in every proper subgame. A partially mixed MPE is defined similarly.

Denote firm i 's equilibrium value at state (s^a, s^b) , V_{s^a, s^b}^i . In a symmetric equilibrium, $V_{s, s'}^a = V_{s', s}^b$. For simplicity of notation when considering symmetric equilibria, we sometimes omit the firms' index.

3. DEFENSIVE PUBLICATIONS

Our goal is to understand the use of publications in an R&D race. Because innovation is evaluated against the state of the prior art, as a result of publications firms will need to achieve more innovation steps. Publication sets back the competitors and prolongs the race. Thus it is costly for both firms. It delays the prize and forces the firms to advance in research more than they would have needed without publications. In our analysis we identify why and when firms would benefit from defensive publications.¹⁹

In this section we assume that the hazard rate is fixed and the race is symmetric. We focus on publication as the choice variable avoiding the extra complexity that an investment decision inflicts. This simplifies the model sufficiently to offer an analytical solution. We show that defensive publications can improve the follower's chances of winning

19. Information disclosure in this paper takes place before the conception of the innovation or the filing of a patent and not through the patent system as the disclosure studied by Anton and Yao (2003).

the race and that due to the tradeoff between this benefit and the cost of publishing, the follower publishes when far enough behind and the leader is close to winning the race.

Let us suppose that having made a sunk investment before the race began, each firm has a constant hazard rate λ_0 . Because the hazard rate is equal for both firms and constant, the firms have equal probabilities to be the first to advance a step. The expected discounted value at (s^a, s^b) is derived in the appendix. It is given by

$$V_{s^a, s^b}^i = \gamma (V_{s^a+1, s^b}^i + V_{s^a, s^b+1}^i), \quad \text{where } \gamma = \frac{\lambda_0}{r + 2\lambda_0}. \quad (1)$$

Proposition 1 shows us that for any patentability requirement of $n \geq 3$ steps, there are parameter values for which publishing research results is in the best interest for firms engaged in a patent race. The incentive to publish arises for a follower. We need at least three steps to have a state in which a follower has something to publish. When $n = 1, 2$, no publication in all states is an equilibrium.

PROPOSITION 1 (Incentive to Publish): *For $n \geq 3$, a Markov perfect equilibrium must involve publications when $n - \frac{1-\gamma}{\gamma^2} > 0$, that is, when r is small enough, when λ_0 is high enough or for a large n .*

Intuitively, publications prolong the race but may improve a follower's probability of catching up with the leader. With a small interest rate or with a large hazard rate, the cost of prolonging the race is small and the benefit for the follower outweighs it.

The costs and benefits of disclosure in this game vary nonlinearly with the stage of the game due to the dynamic nature and set up of the game. When a rival has $n - 1$ steps a transition to a terminal node may occur at any time. In other states, publication can be postponed until the next time the firm would face an immediate risk of losing. In the presence of the tradeoff between the cost and benefit of publications and this nonlinearity, we expect a follower to wait with publications to the last moment (when the leader has attained $n - 1$ steps). In this way the follower can publish when there is risk that the leader might win the race after the next innovation step, but avoid unnecessary publications if he gets lucky.

Patient firms can enjoy the benefit of publication without worrying about prolonging the race. This suggests that the follower will always publish when the leader is a step away from winning the race. We refer to this strategy as the "publish as a last resort" strategy. When $r = 0$, the values of the game in every state are independent of λ_0 . This simplification allows us to find an explicit form for the values of the game under the proposed strategy.

PROPOSITION 2 (Value Function): *When $r = 0$ and the firms choose the “publish as a last resort” strategy the values in every state are*

$$V_{s^i, s^j}^i = \frac{n + s^i - s^j}{2n} v \quad \text{for all } 0 \leq s^i, s^j < n. \quad (2)$$

We use a mathematical induction to prove that without discounting, when the firms use the “publish as a last resort” strategy, $V_{s^a, s^b} = V_{s^a-1, s^b-1}$. Thus, the values depend only on the distance between the two competitors. The follower publishes whenever the leader is close to the patent. Given patient firms, the leader can secure a patent only after the follower has gradually published all his successes.

In Proposition 3 we present the results on equilibrium publishing behavior. For patient firms, we show that “publish as a last resort” is a Markov perfect equilibrium. For impatient firms, we show that the follower is more likely to publish the further behind he is.

PROPOSITION 3 (Equilibrium Publishing Behavior):

1. *There is some \bar{s} such that in equilibrium the follower publishes in states $(s, n - 1)$ if $0 < s < \bar{s}$ and doesn't publish if $s > \bar{s}$.*
2. *When the discount rate is small enough, “publish as a last resort” is a Markov perfect equilibrium.*

Intuitively, this monotonicity result holds true since the follower will avoid publication when his chances of leapfrogging ahead and winning the race are large enough. The closer the follower is to the leader, the higher these chances are. If publication is worthwhile at state $(s, n - 1)$ then this is true also at state $(s - 1, n - 1)$ where catching up with the leader is even less likely.²⁰ For patient firms, we show that publishing as a last resort is an equilibrium by verifying that the equilibrium conditions are met for the values found in Proposition 2. Then, we argue that this result holds in a neighborhood of $r = 0$.

Without a publications fee, multiple equilibria exist because a simultaneous decision on publication implies that when one firm publishes in some state, the other firm is indifferent between publishing and not publishing at that state. Moreover, for patient firms, by Proposition 2, $V_{s^a, s^b} = V_{s^a-1, s^b-1}$ for $0 \leq s^a, s^b < n$; therefore, there are additional payoff equivalent strategies with more publications. The assumption of a small publications fee ensures uniqueness.

20. Note that this monotonicity holds in equilibrium but was not shown to fully characterize it for intermediate values of r and a large n . Numeric solutions for fairly large values of n support the conjecture that there are no leader publications. But the theoretical proof did not rule out the possibility that the leader could have an incentive to narrow the distance between the firms by publishing at a state $(i, 0)$ to maintain a closer race and avoid publications.

PROPOSITION 4 (Uniqueness): *With a small enough r and a small publication fee, publish as a last resort is the unique MPE.*

With a discrete choice to publish or not, a pure strategy equilibrium does not always exist. For intermediate values of the interest rate, the tradeoff between the costs and benefits of publications may only be resolved with mixed strategies. We solve for the partially mixed strategy equilibria in which firms may mix between publishing and not publishing in a race with $n = 3$ innovation steps.

3.1 AN EXAMPLE

In order to find the equilibrium strategy in a model with arbitrary discount rate, we restrict attention to a model where only $n = 3$ innovation steps above the prior state of the art are required for a patent. We expect to find that if publication occurs, it is by the follower at state (1, 2).

When the discount rate is small, there is a pure strategy equilibrium with publications. When it is large, the cost of prolonging the race offsets the benefit from publications and there is a pure strategy equilibrium with no publications. With intermediate discount rates, if no one publishes, the follower has an incentive to publish, but if the follower publishes, the value at state (1, 2) is lowered so that a deviation to no publication in this state is profitable. For these parameter values we find a partially mixed strategy equilibrium in which the follower publishes at state (1, 2) with some probability q .

PROPOSITION 5 (Equilibrium): *For every λ_0 , there exist $0 < \underline{r}_{\lambda_0} < \bar{r}_{\lambda_0} < \infty$ such that the unique symmetric Markov perfect equilibrium strategy is: publish only as a follower in state (1, 2) with probability*

$$q = \begin{cases} 1 & \text{for } 0 \leq r \leq \underline{r}_{\lambda_0} \\ q_\gamma \in (0, 1) & \text{for } \underline{r}_{\lambda_0} < r < \bar{r}_{\lambda_0} \\ 0 & \text{for } \bar{r}_{\lambda_0} \leq r \leq \infty \end{cases},$$

where $q_\gamma = \frac{\gamma + 3\gamma^2 - 1}{\gamma(1 + 3\gamma^3 - 2\gamma)}$ and $\gamma = \frac{\lambda_0}{r + 2\lambda_0}$.

Having found an explicit form for the probability of publication, we can derive comparative static results. We find that the lower r and the higher λ_0 , the larger the probability for follower publication since the cost of publication caused by prolonging the race is relatively low when firms are patient and when research advances rapidly.

4. STRATEGIC PUBLICATIONS IN ASYMMETRIC RACES

This section illustrates the richness of the model in explaining additional incentives to publish, not only by followers. We consider three examples

of asymmetric races in which a firm that has an advantage over its rival would find it profitable to publish.

4.1 PROTECTING A CORE PATENT

Rinner (2003) suggests that a firm holding a core patent has an incentive to publish minor improvements. Publishing protects the core patent by preventing others from patenting an improvement. This business practice has been recommended to companies by intellectual property lawyers and experts.²¹

Assume firm *a* holds a patent and is making a flow profit of $w > 0$ for as long as no subsequent patent is granted. We modify firm *a*'s dynamic programming, equation (1) to incorporate its flow benefit:

$$V_{s^a, s^b}^i = \frac{\lambda_0 V_{s^a+1, s^b}^i + \lambda_0 V_{s^a, s^b+1}^i + w}{2\lambda_0 + r}$$

When $r = 0$, firm *a*, the core patent holder, would find it profitable to publish at state $(n - 1, n - 1)$. By publishing, firm *a* continues to collect the reward from its core patent for a longer time (at least two steps would need to be accomplished instead of just one) without diminishing its probability of eventually winning the new patent (because the state remains symmetric). In addition, when the value of the new patent is small relative to the gain from the existing patent, firm *a* has an incentive to publish as a leader with $n - 1$ steps.²² With a positive discount rate the incumbent would find it profitable to publish when w is large enough to offset the discounting of the prize.

4.2 ASYMMETRIC INVESTMENT COST

When the firms need to pay a fixed flow cost c^i for their investment in research, the more efficient firm can use publications to drive the inefficient firm out of the race. Assume that both firms are patient and have a hazard rate λ_0 , but firm *a* incurs a lower flow cost. For simplicity,

21. See PatentCafe, <http://www.patentcafe.com/corp/pr20001115.asp> and A. G. Schaier, Partner at Carmody & Torrance LLP's <http://www.carmodylaw.com/CM/Articles/art.defensepub.asp>. Note that even if the increments are patentable, sufficient improvement is needed to be detrimental to the core patent holder and of substantial value to its rival.

22. A sufficient condition for firm *a* to have an incentive to publish at state $(n - 1, 0)$ is $v < \frac{(2n-3)w}{2\lambda_0}$ because then, $V_{n-2,0} \geq \frac{(2n-2)w}{2\lambda_0} > \frac{\lambda_0 v + \lambda_0 V_{n-2,0} + w}{2\lambda_0} = V_{n-1,0}^{\text{dont publish}}$. Where the first inequality holds because *a* can prolong the race by at least $n - 2$ steps over the n -step requirement. In a two-stage race firm *a*'s equilibrium strategy is: if $v \leq \frac{2w}{\lambda_0}$ publish at states $(1, 0)$ and $(1, 1)$ and if $v \geq \frac{2w}{\lambda_0}$ publish only at state $(1, 1)$.

$0 = c^a < c^b$.²³ With a high-enough flow cost, firm b does not publish at $(n-1, n-2)$. If firm a doesn't publish at $(n-1, n-1)$, its value is $\frac{v}{2}$, and if it publishes, its value would remain at least $\frac{v}{2}$. For firm b that bears a flow cost, the value at state $(0, 0)$ is $V_{0,0}^b = \frac{v}{2} - TC(0, 0)$. The total cost $TC(0, 0)$ is higher the longer the expected duration of the race. By publishing at $(n-1, n-1)$ firm a prolongs the race and if c^b is high enough, this would increase the cost for firm b and induce it to drop the race.²⁴

4.3 ASYMMETRIC PROBABILITIES OF SUCCESS

Assume now that the two firms differ in their probabilities of success. Suppose the hazard rates for the two firms are $\lambda^a > \lambda^b$. Firms are patient $r = 0$. The strong firm a has an incentive to publish at state $(n-1, n-1)$. At this state, the race ends after one step. Firm a 's probability of success is $\gamma_a = \frac{\lambda^a}{\lambda^a + \lambda^b} > \frac{1}{2}$. If firm a publishes, at the new state $(n-2, n-2)$ the race will end in at least 2 steps (possibly more if equilibrium involves publication). With two steps left a 's probability of success is $\gamma_a^2 + 2\gamma_a^2(1 - \gamma_a) > \gamma_a$. With a higher probability of success firm a increases its probability of winning the race by setting both firms back.²⁵

5. PUBLICATIONS AND R&D INVESTMENT

In this section we combine the standard consideration of strategic R&D investments in an uncertain race with the new feature, publication. We show that for an open set of parameter values the Markov perfect equilibrium must involve publications. Publications may substitute effort when the marginal cost of investment is high. The model in this section is closely related to the models by Grossman and Shapiro (1987) and by Harris and Vickers (1987) in which the hazard rates depend on the intensity of investment. In these papers, each player is striving to reach a given finish line before the rival.²⁶ In contrast to their work, the

23. Firm b 's dynamic programming equation looks just like that in the previous subsection with $w = -c^b$.

24. The MPE for $n = 2$ is

If $\frac{\lambda_0 v^b}{3} > c^b > \frac{\lambda_0 v^b}{6}$, Firm a : Publish at state $(1, 1)$ only. Firm b : Never publish. Drop out at state $(1, 0)$.

If $\frac{\lambda_0 v^b}{3} < c^b < \frac{3\lambda_0 v^b}{7}$, Firm a : Publish at state $(1, 1)$ only. Firm b : Stay out.

25. In a two-stage race, the equilibrium strategy is: firm a publishes at state $(1, 1)$ only, firm b doesn't publish.

26. They focus on how the efforts of competitors in a race vary with the intensity of rivalry. A main result in their paper as well as in Grossman and Shapiro (1987) is that the leader invests more than the follower. This continues to hold in the publications model.

finish line in our model is endogenous since the prior art can be changed by publications.

Harris and Vickers (1987) acknowledged the apparent difficulty in obtaining analytical results in the no publications model. The consideration of strategic publications in addition to the investment decision adds a significant complexity to the model. Although the dynamics of the patent race when no publication is allowed is such that each state is only visited once, in the publication model states may be revisited. In the no publication model, the values can be calculated from state $(n - 1, n - 1)$ backward. In the publications model, one can no longer do so. We use the results from the simplified fixed hazard rate model to identify the incentive to publish in the race with investment. To allow this analysis, we take a cost function of the form

$$c(\lambda) = \begin{cases} c_0 C(\lambda - \lambda_0) & \text{for } \lambda \geq \lambda_0 \\ 0 & \text{for } \lambda < \lambda_0 \end{cases} \tag{3}$$

where $c_0 > 0, \lambda_0 > 0$, and the function $C(x)$ is strictly increasing and convex satisfying: $C(0) = 0, C'(0) = 0, C'(x) \rightarrow \infty$ as $x \rightarrow \infty$ and $C''(x) > 0$.²⁷ We interpret λ_0 as a fixed hazard rate that each firm has due to its fixed investment. Investment increases the hazard rate, λ above the fixed hazard rate λ_0 , and has decreasing returns.

In equilibrium, if no publication occurs at state (s^i, s^j) the value function as derived in the Appendix is given by

$$V_{s^i, s^j} = \frac{\lambda_{s^i, s^j} V_{s^i+1, s^j} + \lambda_{s^j, s^i} V_{s^i, s^j+1} - c(\lambda_{s^i, s^j})}{r + \lambda_{s^i, s^j} + \lambda_{s^j, s^i}} \tag{4}$$

A hazard rate $\lambda_{s^i, s^j} > \lambda_0$ is optimal if and only if it satisfies the first-order condition derived from equation (4),

$$V_{s^i+1, s^j} - V_{s^i, s^j} = c'(\lambda_{s^i, s^j}) \tag{5}$$

That is, in every state with investment, the marginal cost effort equals its marginal benefit. As c_0 goes to infinity, the model approaches the fixed hazard rate model. This ensures the use of defensive publications on a large range of parameter values.

PROPOSITION 6 (Incentive to Publish in Investment Race): *The MPE must involve publications when $n - \frac{1-\gamma}{\gamma^2} > 0$ for a sufficiently large c_0 .*

Recall that by Proposition 1, $n - \frac{1-\gamma}{\gamma^2} > 0$, where $\gamma = \frac{\lambda_0}{r + 2\lambda_0}$ is the condition for an equilibrium with publications in the fixed hazard

27. Taking a cost function of form (3) is convenient and less restrictive than it may seem. For an equilibrium with all $\lambda_s > \lambda_0$, one can also find a cost function that is strictly increasing for all $\lambda > 0$, and has the same equilibrium.

rate model. This condition holds true when, for example, there is no discounting. As $c_0 \rightarrow \infty$, the values of the game converge to the values in the fixed hazard rate model. Thus, the follower has an incentive to publish when the leader is close to the end of the race. Intuitively, followers can increase their chances of catching up either by putting in more effort or by publishing. Thus, defensive publications can substitute for investment and are therefore more likely when the marginal cost of R&D is high. As in the fixed hazard rate model, because publications prolong the race, they are less costly when the firms are patient and when the success probability is high.

COROLLARY 1: *When $n - \frac{1-\gamma}{\gamma^2} > 0$ for a sufficiently large c_0 publications prolong the race.*

When the probability of success is fixed and the race involves publications, the expected duration of the race must be longer than in a race with no publications since the number of steps achieved is larger. In the investment race, high investments can expedite innovation. However, when the cost parameter increases, the duration of the race must be close to that in the fixed hazard rate model and therefore the race with publications is longer.

5.1 AN EXAMPLE

We consider the race with $n = 3$ requirement, patient firms, and a quadratic cost function. In this case, the marginal cost is linear, simplifying computation of equilibria. Varying parameter values, we numerically compute the equilibrium. The results are summarized in the following claims.

CLAIM 1: For every $\lambda_0 > 0$, there exist $0 < \underline{c}_{\lambda_0} < \bar{c}_{\lambda_0} < \infty$ such that the unique symmetric Markov perfect equilibrium strategy involves follower publication at state (1, 2) with probability

$$\begin{cases} q = 0 & \text{for } 0 < c_0 \leq \underline{c}_{\lambda_0} \\ q(c_0) \in (0, 1) & \text{for } \underline{c}_{\lambda_0} < c_0 < \bar{c}_{\lambda_0} \\ q = 1 & \text{for } \bar{c}_{\lambda_0} \leq c_0 < \infty \end{cases} .$$

The probability of publication increases with the cost parameter c_0 . Thus publications substitute for investment.

Grossman and Shapiro (1987) and Harris and Vickers (1987) find that in a no publications equilibrium of this model, the leader invests more than the follower. Proposition 7 shows that this result is true in the

publications equilibrium as well. When the follower publishes at state (1, 2) the leader makes a greater effort than the follower.

PROPOSITION 7 (Leader Invests More than Follower): *If equilibrium involves publication at state (1, 2), then for $s^i > s^j$, $\lambda_{s^i, s^j}^i > \lambda_{s^i, s^j}^j$.*

The expected payoff at the initial state (0, 0) determines the fixed R&D investment a firm would be willing to make in order to compete for the prize. The value at the initial state (0, 0) is lower in the game with publications compared to a race with no publications. This lower value is a result of a higher R&D expenditure during the race in the publications model where more innovation steps need to be achieved in order to secure a patent. This finding suggests that the incentive to enter the innovation race is lower when strategic disclosure is an available strategy.

CLAIM 2: The value V_{00} is lower in the model with publications.

In state (2, 0), in the publications model, if the leader is the first to complete the next step, she secures the patent. If the follower advances first, not only will the gap narrow but the follower can also publish. Therefore, we expect both players to invest more compared to a no publications model.

CLAIM 3:

1. The flows of effort $\lambda_{0,2}$ and $\lambda_{2,0}$ are higher in the model with publications.
2. In all other states, λ_s is at least as high in the no publications model.

Although the three-step model only involves follower publications, in a race with a strong patentability requirement a leader may also have an incentive to publish. In the investment race, if $\lambda_0 = 0$, and a small number of innovation steps are required, the no publication strategy is an equilibrium. The follower's ability to adjust investment eliminates his incentive to publish. However, when the patentability requirement is high enough ($n \geq 6$, according to numeric computations), the equilibrium must involve strategic publications. The distance from the end of the race has a complex cyclical behavior (see Harris and Vickers, 1987). In this case, a leader may publish preemptively. Publication allows the leader to avoid states with fierce competition.

6. RELATED LITERATURE

Parchomovsky (2000) drew attention to the fact that patentability is a function of the state of the prior art, which gives firms the power to affect

the patentability of a rival's invention by publishing. Parchomovsky argues that publications can preempt patenting future discoveries and this motivates a firm that is losing a race to publish. Lichtman et al. (2000) challenge and expand the work of Parchomovsky. Both papers discuss the role of the prior art in the patent system and the statutory provisions relevant to the strategic use of publications. According to Lichtman et al. (2000) a follower's incentive to publish is driven by the ability to compete in a subset market based on the information disclosed. However, as the authors note, since the laggard lacks intellectual property rights on the disclosed information this value is likely to be very limited, leaving the laggard little incentive to publish. This paper shows how even in the absence of any direct profit from publications followers have a strong incentive to publish if by doing so, they can prolong an innovation race.

In concurrent independent work, Baker and Mezzetti (2005) also investigate strategic disclosure. The premise for both papers is that publications increase the state of the prior art, prolong the race, and make patenting more difficult. However, the papers explore the use of strategic disclosure in different environments, and hence reach different conclusions. In Baker and Mezzetti a race between firms M and N takes place in three periods. In period 1, only firm M can conduct research and disclose. In period 2, only firm N can conduct research and disclose, and in the final period, only firm M moves again. Throughout period 2, in which firm N conducts research, obtains results, and decides on disclosure, its rival cannot act. In particular, regardless of how close firm M is to the finish line, its probability of winning in period 2 is zero. Thus, their results are most applicable for situations in which a firm has achieved some nonpatentable progress but expects a delay before it can begin pursuing the next milestone. A continuous-time, simultaneous-move game as in our paper is, we believe, a more natural way to model an R&D race. Here both competitors can revise their actions at any stage of the ongoing race.

Baker and Mezzetti's (2005) first proposition states: "The disclosing firm's optimal level of disclosure increases with m , the knowledge of the disclosing firm. This is true no matter whether the disclosing firm leads or trails in the race." This result, which contradicts ours (see Propositions 1 and 3) seems to depend on the fact that in period 2 firm M has a zero probability of success. Consider, for example, a situation in which M is very close to winning the race, and N is behind. According to Baker and Mezzetti the leader M would publish to make it more difficult for N to win in period 2, when only N could win. In our model, the leader has a high probability of winning at any time. Hence, it has no incentive to publish in this situation. In Baker and Mezzetti the follower

N would not publish in this situation since the game ends in period 3. In our model the follower, taking into account that the leader is close to winning the race, would gain from publishing.

In their introduction, Baker and Mezzetti (2005) point out that there are differences between leaders' and laggards' motives for publication. For a firm trailing the race "better chances to catch up" and for leaders "extending the costs of racing that will in certain instances discourage the laggard from racing so aggressively." However, in their analysis, whether a firm leads or lags seems to play no role. They note that "none of the above intuitions requires any specific assumption as to whether the disclosing firm leads or lags in the race. Thus all of the propositions explicitly apply to both leader and laggard firms." In fact, since they analyze a three-period sequential game, it is always firm *M* (whether leader or follower) that may publish and only in period 1. Its rival firm *N* can't benefit from publishing since this is its last move before the game ends. The infinite horizon dynamic model in this paper makes it possible to differentiate the behavior of leaders and followers as the race progresses. Firms' relative position plays a significant role resulting in the publish-as-a-last resort strategy where followers publish.

In both papers there is uncertainty with respect to innovation. This paper assumes a Poisson process (to make the dynamic game tractable). Baker and Mezzetti's (2005) assumptions on the stochastic process are less restrictive. In Baker and Mezzetti information is a continuous variable thus even an infinitesimal level of disclosure raises the patentability bar. In this paper, research progresses in discrete steps so only sufficient disclosure affects patentability.

When extending the model to include investment, their work focuses on comparing the amount of information disclosed when investment is and isn't possible. This paper focuses on different issues. We maintain assumptions commonly made in the economic literature on patent races allowing a comparison with the standard literature. We show that publishing can substitute investment as the marginal costs of investment rise.

7. CONCLUDING REMARKS

Scotchmer and Green (1990) explain that the social goal of protecting profits is served by a strong novelty requirement while the goal of disclosure is served by a weak novelty requirement. O'Donoghue (1998) also suggests that a strong patentability requirement can stimulate R&D investment and enhance welfare. We showed that a strong patentability requirement may give rise to defensive publications, so that contradicting welfare effects are present: on the one hand, information disclosure,

free of market power; on the other hand, possibly less incentive to invest and more inefficient duplication.

In interpreting n as a patentability requirement, we have implicitly assumed that fewer than n steps (the disclosed information) couldn't be patented. An innovation step might not be patentable if it isn't a sufficient improvement,²⁸ or if it isn't by itself useful.²⁹ The current US system is often criticized for granting patents too easily. But recently there have been calls to strengthen the patentability requirements.³⁰ This in terms of our model increases n and could give rise to more defensive publications. Even with a weak patentability requirement, the intuition in this paper can have important implications. Leading firms might prefer not to patent interim technologies³¹ but the threat of follower's publications might induce leaders to do so.

The model and intuitions may be applicable to any race in which multiple steps must be accomplished in order to win a prize, and competitors can use disclosure to prolong the race. Even if a firm's goal is not to patent an innovation, they likely have the goal of getting "credit" for the work (media attention, professional recognition, etc.) and if to get credit you must have a sufficient advance beyond what's already in the public domain, then the basic ideas of the paper would still hold. Consider the following example: two companies share a market for some service and are making zero profits. There is a cost to consumers in switching service providers, thus a firm could capture the whole market only if it offers a sufficiently improved service (n steps). Smaller improvements would not make consumers switch, but can raise the bar. If a leader is close to introducing n steps, the follower would (if he can) defensively improve by a step. The method and intuitions we developed apply also to other versions of our model: one in which any published step sets both competitors back to the beginning, and a new innovation race begins; or one where a minimum of k steps disclosure is needed to raise the bar.

Following are some limitations of the model. In order to have a Markov structure in which firms' strategic behavior depends only on the two-dimensional state (s^a, s^b) , we assumed that the prize and the costs

28. When the European Patent Office revoked Myriad Genetics' patent on a breast cancer genetic test they said in a news release that "... the work mentioned in the patent was not inventive enough to qualify for patent protection." See Andrew Pollack. "Patent for US Genetic Test Is Revoked in Europe." *TechNewsWorld*. Global. 24 May, 04.

29. In January 2001 the US PTO issued new guidelines aimed at stopping companies from patenting genes before they have any use for them. See *The New York Times*. Business/Financial Desk. Section C; Col. 5; Pg. 3. 6 January, 2001.

30. The Federal Trade Commission recommended strengthening the "obvious" standard and the National Research Council recommended "reinvigorating the nonobviousness standard." See *NY Law Journal*. May 26, 2004, News; Vol. 101; Pg. 3.

31. See, for example, Scotchmer and Green (1990).

remain constant even if the race is prolonged by publications. In reality, these parameters may change depending on the prior art or progress. A patent may become more valuable with progress. Diminishing returns to R&D would suggest a higher cost the more steps are completed while experience may lower the cost. If research becomes more difficult, publication may become less attractive and vice versa.

In this paper, the firms' progress is common knowledge. Firms sometimes obtain information about their rival's research progress, but some level of asymmetric information likely exists. Asymmetric information would be significantly more difficult to analyze since strategies would depend on beliefs that change over time. It also adds a risk of revealing information that the rival does not already possess. We may expect strategic publications when the firm's belief that the rival is close to winning is high.

The possibility that firms could negotiate and agree to license the technology in return for no disclosure is left for future research. We only note that for the leader it's best if negotiations take long enough for her to complete the last step. Thus it may be difficult to reach an agreement.

We modeled research advances as discrete steps. This analogy is commonly made in economic modeling of R&D. Steps may be defined by the nature of the technology or by the way people perceive progress and measure it (in units of size, dollars saved, etc.). Firms might make continuous progress but clearly notice it and be able to articulate it only in certain natural milestones.³² We assume such milestones exist and are not arbitrarily defined. The crucial aspects of our discreteness assumption are that: (i) in order to affect the patentability a noninfinitesimal amount of disclosure is needed;³³ (ii) the n th step may be achieved in a sudden leap. If the follower could always wait a bit longer and publish a bit less the results of our model would not hold. Publish as a last resort could loosely be understood as followers benefit from defensively publishing; they prefer to disclose the least amount that would affect patentability, when the leader might suddenly win the race.

The introduction described some evidence of defensive publications. Competing theories for why firms publish exist and it's difficult to identify empirically their motive for several reasons such as: R&D practices are not easily observed (e.g., which firms are competing, what their relative positions are); firms may prefer to claim to publish for the sake of a noble cause such as making a technology available and free

32. In writing a proof for a theorem, for example, many trials and errors could be thought of as continuous progress. Alternatively, only once a proof is established can it be perceived as an accomplished research step.

33. Although any level of progress can be published, it seems that in the patent system only a sufficient amount of disclosure would raise the hurdle for others.

to all rather than admitting to publish strategically; if a firm publishes and raises the prior art, its rival would continue research until it makes sufficient progress making it hard to observe that a publication preempted patent or prolonged a race; If the patentability requirement is weak and almost any small improvement can be patented there is less room for strategic publications. Only in recent years has there been some change toward a stronger patentability requirement. This recent trend and call for strengthening the patentability requirement makes it particularly interesting to understand strategic disclosure but also means that some cases may not yet be documented.

APPENDIX

In the first part of the Appendix, we derive the dynamic programming equations. The second part of the Appendix contains all the proofs.³⁴

A.1 THE DYNAMIC PROGRAMMING EQUATIONS

We formulate the dynamic programming equations making no assumptions on the interest rate r and the cost function $c(\lambda)$. When each firm i exerts effort that yields a hazard rate λ_{s^i, s^j}^i the expected payoff V_{s^i, s^j}^i satisfies the dynamic programming equation

$$rV_{s^i, s^j}^i = \lambda_{s^i, s^j}^i (V_{s^{i+1}, s^j}^i - V_{s^i, s^j}^i) + \lambda_{s^i, s^j}^j (V_{s^i, s^{j+1}}^i - V_{s^i, s^j}^i) - c(\lambda_{s^i, s^j}^i).$$

That is, the rate of return equals the expected gains and losses minus the flow cost. In a symmetric equilibrium $\lambda_{s^i, s^j}^j = \lambda_{s^j, s^i}^i$. Insert λ_{s^i, s^j}^j , drop the firm index, and solve for the expected value at (s^i, s^j) to obtain

$$V_{s^i, s^j} = \frac{\lambda_{s^i, s^j} V_{s^{i+1}, s^j} + \lambda_{s^j, s^i} V_{s^i, s^{j+1}} - c(\lambda_{s^i, s^j})}{r + \lambda_{s^i, s^j} + \lambda_{s^j, s^i}}. \tag{A1}$$

When at least one of the firms achieved n innovation steps, the patent is granted. The values at such terminal nodes are given by

$$V_{n, s^j}^i = v, \quad V_{n, s^j}^j = 0 \quad \text{for} \quad 0 \leq s^j \leq n - 1. \tag{A2}$$

If either firm publishes $k \leq \min(s^a, s^b)$ steps at state (s^a, s^b) , the values at that state will satisfy

$$V_{s^a, s^b}^i = V_{s^a - k, s^b - k}^i \quad \text{for} \quad i = a, b. \tag{A3}$$

34. A more detailed version of the proofs can be found at <http://www.arts.cornell.edu/econ/bar/>.

Fixing firm j 's strategy, i chooses the maximum between V_{s^i-1,s^j-1}^i if he publishes and the value (A1) with an optimal choice of λ_{s^i,s^j} if he doesn't publish.

A.2 PROOFS

Proof of Proposition 1. The equations (1) for all (s^i, s^j) define the value function when firms never publish. Not publishing is a MPE when

$$V_{s^i,s^j}^i \geq V_{s^i-1,s^j-1}^i \quad \text{and} \quad V_{s^i,0}^i \geq V_{s^i-1,0}^i \quad \text{for all} \quad 1 \leq s^i, s^j \leq n-1.$$

Else, there is an incentive to publish. However, we show that under some conditions on the parameters n, r and λ_0 , the inequality

$$V_{s^i,n-1}^i < V_{s^i-1,n-2}^i \tag{A4}$$

holds giving the follower an incentive to publish. Let us write the inequality (A4) as a function of the parameters only. From (1) we know that $V_{s^i,n-1}^i = \gamma V_{s^i+1,n-1}^i$. Recursively using (1) we obtain

$$V_{s^i,n-1}^i = \gamma^{n-s^i-1} V_{n-1,n-1}^i = \gamma^{n-s^i} v. \tag{A5}$$

Substitute (A5) in the left-hand side of (A4) and substitute (1) in the right-hand side to obtain

$$\gamma^{n-s^i} v < \gamma (V_{s^i,n-2}^i + V_{s^i-1,n-1}^i).$$

Continue substituting (A5) and (1) and rearranging to find that (A4) holds if and only if

$$s^i < n + 1 - \frac{1 - \gamma}{\gamma^2}. \tag{A6}$$

The right-hand side of (A6) is an *explicit* cutoff point s^* between having an incentive to publish or not at states $(s^i, n-1)$ when the strategies used are never to publish. When $s^* > 1$, the follower at state $(1, n-1)$ prefers publishing. $s^*(n, \gamma)$ is increasing in n and γ . If $n \geq 3$ with large enough γ , in equilibrium, the firms must publish. Hence, publications occur with large λ_0 and with small r . □

Proof of Proposition 2. We show this in 3 steps.

Step 1. We show that

$$V_{s,n-1} = \dots = V_{s-k,n-1-k} \dots = V_{0,n-1-s}.$$

When $r = 0$ and $c = 0$, $V_{s,s^i}^a + V_{s,s^i}^b = v$. Thus, in a symmetric equilibrium, $V_{s,s} = \frac{v}{2}$. Because the follower publishes at state $(s, n-1)$,

according to equation (A3), $V_{s,n-1} = V_{s-1,n-2}$. At state $(n-3, n-2)$ the firms do not publish. Therefore, the value is:

$$\begin{aligned} V_{n-3,n-2} &= \frac{1}{2}(V_{n-2,n-2} + V_{n-3,n-1}) \\ &= \frac{1}{2}(V_{n-3,n-3} + V_{n-4,n-2}) = V_{n-4,n-3}. \end{aligned}$$

Next, we show by induction that $V_{s-1,n-2} = V_{s-2,n-3}$, for all $0 < s < n-2$. Furthermore, for all s^b and $0 < s^a < s^b$, $V_{s^a,s^b} = V_{s^a-1,s^b-1}$. We just proved this result for $s^b = n-2$. Using two nested backward inductions we now prove that the equality $V_{s^a,s^b} = V_{s^a-1,s^b-1}$ holds for all s^b and s^a , $0 < s^a < s^b$. Because $V_{s^b,s^a} = v - V_{s^a,s^b}$ the equality holds for all (s^a, s^b) .

Step 2. We show that for all s ,

$$V_{0,s} = \frac{n-s}{2n}v \quad \text{and} \quad V_{s,0} = \frac{n+s}{2n}v.$$

From equation (1) and Step 1 it follows that

$$V_{0,s} = \frac{1}{2}(V_{1,s} + V_{0,s+1}) = \frac{1}{2}(V_{0,s-1} + V_{0,s+1}) \quad \text{for all } 0 < s < n,$$

where $V_{0,n} = 0$ and $V_{0,0} = \frac{1}{2}v$. It is easy to verify that the unique solution to this system is $V_{0,s} = \frac{n-s}{2n}v$. Given these values we can find $V_{s,0} = v - V_{0,s}$.

Step 3. We show that for all s ,

$$V_{s^a,s^b} = \frac{n+s^a-s^b}{2n}v \quad \text{for all } 0 \leq s^a, s^b < n.$$

For $s^a \geq s^b$, by Step 1, $V_{s^a,s^b} = V_{s^a-s^b,0}$ and from Step 2, $V_{s^a-s^b,0} = \frac{n+s^a-s^b}{2n}v$. For $s^a < s^b$, by Step 1, $V_{s^a,s^b} = V_{0,s^b-s^a}$ and from Step 2, $V_{0,s^b-s^a} = \frac{n+s^a-s^b}{2n}v$. \square

Proof of Proposition 3.

1. We show that if in equilibrium the follower publishes at $(s, n-1)$, for $s > 1$, with some positive probability, then the follower also publishes at $(s-1, n-1)$ with probability 1. That is, we need to show that if the firm publishes at $(s, n-1)$, then

$$\gamma V_{s,n-1}^i < V_{s-2,n-2}^i \quad (\text{A7})$$

where the left-hand side is the value for the follower, at state $(s-1, n-1)$ if the firm does not publish and the right-hand side is the value if the firm publishes. The firm publishes at $(s, n-1)$,

$V_{s,n-1}^i = V_{s-1,n-2}^i$. In addition, for the follower:

$$\gamma(V_{s-1,n-2}^i + V_{s-2,n-1}^i) \leq V_{s-2,n-2}^i.$$

Thus, (A7) holds if

$$\gamma V_{s-1,n-2}^i < \gamma(V_{s-1,n-2}^i + V_{s-2,n-1}^i).$$

Or $0 < \gamma V_{s-2,n-1}^i$, which must be true.

2. It is easy to verify that the values $V_{s^a,s^b} = \frac{n+s^a-s^b}{2n}v$ satisfy the inequalities conditions

$$V_{s^a,s^b} \geq V_{s^a-1,s^b-1} \quad \text{for all } 0 < s^a, s^b < n, \tag{A8}$$

$$V_{s,0} \geq V_{s-1,0} \quad \text{for all } 0 < s < n, \tag{A9}$$

$$V_{s,n-1} \geq \gamma V_{s+1,n-1} \quad \text{for all } s < n-1. \tag{A10}$$

It can be verified that the solution to the system with a small-enough discount rate and cost also satisfies (A8). Hence the strategy is a MPE. \square

Proof of Proposition 4. Let ε be a small publication fee for each step published, at any state. We first show that in equilibrium, the leader strictly prefers not to publish at states $(n-1, s)$ and $(s, 0)$. And that the follower strictly prefers to publish at state $(s, n-1)$. For $\varepsilon = 0$, at state $(n-1, 0)$, the value for the leader if she does not publish is

$$\frac{1}{2}(v + V_{n-1,1}) \geq \frac{1}{2}(v + V_{n-2,0}) > V_{n-2,0}.$$

Therefore, $V_{n-1,0} > V_{n-2,0}$ and the leader does not publish. By induction, we show that for all s $V_{s,0} > V_{s-1,0}$. This inequality is strict and would also hold with a small publication fee. The leader at state $(n-1, s)$ will not publish either. If the follower publishes, then the leader is better off avoiding the publication fee. If the follower does not publish, then the leader's payoff if she does not publish is

$$\frac{1}{2}(v + V_{n-1,s+1}) > \frac{1}{2}(V_{n-k,s-k} + V_{n-1-k,s+1-k}),$$

which is the payoff if she publishes and $\varepsilon = 0$. Again, this strict inequality would also hold with a small publication fee.

The follower at state $(s, n-1)$ will necessarily have an incentive to publish. Consider $\varepsilon = 0$. First, we show this for $s = n-2$. The follower will publish in state $(n-2, n-1)$ if

$$\frac{1}{2}V_{n-1,n-1} < V_{n-3,n-2}. \quad (\text{A11})$$

The value in state $(n - 3, n - 2)$ is given by (1), substituting it we obtain the condition

$$\frac{1}{2}V_{n-1,n-1} < \frac{1}{2}(V_{n-2,n-2} + V_{n-3,n-1}).$$

When $r = 0$, $V_{n-1,n-1} = V_{n-2,n-2} = \frac{v}{2}$. Therefore, the inequality (A11) becomes $0 < \frac{1}{2}V_{n-3,n-1}$, which is true. It now follows that the follower will publish at all other states $(s, n - 1)$ (by the proof of Proposition 3). This strong incentive to publish would remain with a small enough publication fee.

We have shown that, the leader strictly prefers not to publish at states $(n - 1, s)$ and $(s, 0)$, and that the follower strictly prefers to publish at state $(s, n - 1)$. When $\varepsilon = 0$, any pair of Markov strategies for which these properties hold, results in the values given in (2). With a small publication fee, the value at state (s^a, s^b) would have an expected benefit as in (2) minus the expected publication expenses. If there exists another equilibrium, firms publish more than in the proposed equilibrium. Therefore, at some state for some firm, the expected expenses would be higher. A deviation from its strategy to the proposed equilibrium strategy would yield the same benefit and a lower expenditure on publication and therefore at this state it would be profitable.

Contradiction. Equilibrium must be unique when there is a small publication fee. Finally, uniqueness holds in a neighborhood of $r = 0$. As $r \rightarrow 0$, $\gamma = \frac{\lambda_0}{r + 2\lambda_0} \rightarrow \frac{1}{2}$. And we note that the inequalities involving $\gamma = \frac{1}{2}$ in the proof are strict and remain true in a neighborhood of $\gamma = \frac{1}{2}$. \square

Proof of Proposition 5. We solve for the values of the game when the probability of publication at state $(1, 2)$ is $q \in [0, 1]$. We check conditions (A8)–(A10) to find the parameter values for which each possible strategy is an equilibrium. The equations depend on γ and not on λ_0 and r independently. The solution obtained is

$$q = \begin{cases} 0 & \text{for } \gamma \leq -\frac{1}{6} + \frac{1}{6}\sqrt{13} \\ \frac{\gamma + 3\gamma^2 - 1}{\gamma(1 + 3\gamma^3 - 2\gamma)} & \text{for } -\frac{1}{6} + \frac{1}{6}\sqrt{13} < \gamma < \frac{1}{6}\sqrt{(30 - 6\sqrt{13})} \\ 1 & \text{for } \frac{1}{6}\sqrt{(30 - 6\sqrt{13})} \leq \gamma \end{cases}$$

The parameter $\gamma = \frac{\lambda_0}{r+2\lambda_0}$, is monotone in λ_0, r . In terms of r we find that $\overline{r}_{\lambda_0} = 2\lambda_0 \frac{4-\sqrt{13}}{-1+\sqrt{13}}$ and $\underline{r}_{\lambda_0} = 2\lambda_0 \frac{3-\sqrt{(30-6\sqrt{13})}}{\sqrt{(30-6\sqrt{13})}}$. \square

Proof of Proposition 6. For all $s, s', V_{s'} - V_s \leq v$. So, by (5) $c'(\lambda_s) \leq v$. When $c_0 \rightarrow \infty$, it must be that the optimal hazard rate at each state approaches $\lambda_0, \lambda_{s,s'}(c_0) \rightarrow \lambda_0$. Otherwise, the marginal cost cannot be bounded, because $C'(x) > 0$ for all $x > 0$. Moreover, the cost function $c(\lambda)$ at the optimal equilibrium investments must approach zero as $c_0 \rightarrow \infty$. Otherwise, for high-enough c_0 the firm can do better by choosing a corner solution $\lambda = \lambda_0$ for which $c(\lambda_0) = 0$. Because equilibrium hazard rates approach the fixed hazard rate λ_0 and the cost approaches zero, the fixed probability race values are the limit of the equilibrium values as the cost parameter $c_0 \rightarrow \infty$. \square

Proof of Proposition 7. Use (4) to obtain

$$v > V_{2,0} + V_{0,2} > V_{1,0} + V_{0,1} > 2V_{1,1}, \tag{A12}$$

where the right inequality follows from specializing (4) to state (1, 1) and the middle inequality follows from specializing (4) to states (1, 0) and (0, 1) and adding up.

Insert the optimal effort from the first-order condition (5) into the dynamic programming equation (4) and rearrange to obtain

$$\begin{aligned} & (V_{s^j+1,s^i} - V_{s^j,s^i})(V_{s^i,s^j} - V_{s^i,s^j+1}) + 2c_0\lambda_0(V_{s^i,s^j} - V_{s^i,s^j+1}) \\ &= \frac{1}{2}(V_{s^i+1,s^j} - V_{s^i,s^j})^2 + 2c_0\lambda_0(V_{s^i+1,s^j} - V_{s^i,s^j}). \end{aligned} \tag{A13}$$

From the first-order condition

$$\lambda_{1,0} > \lambda_{0,1} \iff V_{2,0} - V_{1,0} > V_{1,1} - V_{0,1}.$$

Assume by contradiction that $(V_{1,1} - V_{0,1}) \geq (V_{2,0} - V_{1,0})$. By equation (A13) specialized to states (1, 0) and (0, 1), if $(V_{1,0} - V_{1,1}) > (V_{0,1} - V_{0,2})$ then $(V_{1,1} - V_{0,1}) < (V_{2,0} - V_{1,0})$, which is a contradiction. If, on the other hand $(V_{0,1} - V_{0,2}) \geq (V_{1,0} - V_{1,1})$ then

$$(V_{1,1} - V_{0,1}) \geq (V_{2,0} - V_{1,0}) \geq (V_{0,1} - V_{0,2}) \geq (V_{1,0} - V_{1,1}),$$

where the middle inequality holds by (A12). It follows that $2V_{1,1} \geq V_{1,0} + V_{0,1}$, which is a contradiction to (A12). We conclude that $(V_{1,1} - V_{0,1}) < (V_{2,0} - V_{1,0})$ and the leader at (1, 0) invests more than the follower. A similar argument shows that the leader at (2, 0) invests more than the follower. \square

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